

# AUCTION STRATEGIES FOR REAL WORLD MARKETS

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## AUCTION STRATEGIES FOR REAL WORLD MARKETS

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The theoretical and empirical study of auctions is of importance to economists as it provides unique insights into the behaviors and decision-making processes involved in the setting of prices that are otherwise unobserved in traditional fixed price sales. The papers in this dissertation look at observed outcomes from real world auctions and draw inferences as to the underlying economic behavior.

The first paper examines the practice of auctioneers “fishing” for an opening bid by calling out lower and lower amounts until an opening bid is eventually proffered. I incorporate such a tactic into an auctioneer’s strategy set within a game-theoretic model in which an indivisible good is sold via English ascending-price auction by a seller or auctioneer that cannot commit to a predetermined sequence of starting prices in advance. The analysis departs from previous literature by showing that the English auction is not strategically equivalent to the second-price auction within the dynamic setting. This difference has implications for the optimal starting price path, giving rise to an initial starting price consistent with the Coase conjecture. Additional price dynamics resembling “auction fever” are rationalized within this rational framework.

In the second paper, I, along with a co-author, investigate theories of non-standard preferences and irrational bidding that have been used to explain the behavior of bidders in auctions online. We test these theories using data from a field experiment

that we ran on eBay and supplemented with an observational dataset we collected from eBay. We find little evidence that several of these behavioral mechanisms are important in the field, and instead find behavior consistent with a standard rational model.

The third paper examines the practice of shill bidding, whereby a seller in an online auction bids on his own item. To incorporate shill bidding into a seller's strategy set, I model the sale of a common value item via auction where some proportion of potential buyers have superior information and some proportion of potential sellers participate in the market only for the purpose of fraudulent selling. The model establishes the conditions under which shill bidding is supported in equilibrium.

## BIOGRAPHICAL SKETCH

Born and raised in the sunny splendor of Garden Grove, California, Joe enjoyed a childhood full of outdoor activities, most notably baseball. Although schoolwork came easily to Joe, he was not an A student, having decided at the age of six that he would have to pace himself if he were to continue schooling through the age of 22, when he expected to graduate college. In place of homework, Joe spent his evenings indulging his interest in foreign countries and world cultures by memorizing the globe that his parents had given him for his seventh birthday and reading from his family's World Book encyclopedias. Eventually, a senior-year economics class provided the spark that would ignite Joe's interest in the subject matter of this dissertation.

Following high school in the south suburbs of Chicago, where his family had inconveniently moved at the start of Joe's sophomore year, Joe matriculated to Bradley University in Peoria, Illinois. At Bradley, Joe enjoyed the freedom of college life and blossomed as a student. Majoring in both economics and mathematics, Joe graduated with high honors which earned him a Sage Fellowship upon his entry into the Ph.D. program in economics at Cornell University in Ithaca, NY. While Joe's academic accomplishments are evident, his greatest accomplishment at Cornell was meeting a fellow graduate student, Dana Felice, and convincing her to go out with him. Joe and Dana married after graduation and settled in Chicago, Illinois.

*To my parents, Dalia and Shalom, for your love and encouragement*

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# CHAPTER 1

## SEQUENTIAL ENGLISH AUCTIONS

### 1 Introduction

In his 1967 survey of all things auction, Cassady describes the sale of a range of items from livestock to antiques in which the auctioneer goes *fishing* for an opening bid. The auctioneer begins the proceedings by soliciting bids from the set of buyers at some announced opening bid. If a bid is proffered, the auction proceeds as in typical English ascending-price fashion—soliciting higher and higher bids until the item is hammered down with a gavel. But should the auctioneer fail to find a bidder, the opening bid is reduced and a second attempt is made. This process continues until an opening bid is arrived at which eventually gets the bidding started. Cassady points out a paradoxical outcome of this process: once an opening bid is proffered and the bidding gets under way, it is not uncommon for the bidding to progress beyond the amount of the original opening bid.<sup>1</sup>

It is surprising, given the breadth of the literature on optimal auctions, that the tactic of opening bid fishing has not received more attention. Further, the outcome in which the end price exceeds the amount of the initial starting price is paradoxical in light of existing results. To explain this paradoxical outcome, I model the sale of a single item to a set of buyers in which the seller (or auctioneer whose incentives are aligned with those of the seller) runs an English ascending-price auction (“APA”) with an announced starting price, in each period of an infinite-horizon game. The model considered herein is identical to that of McAfee and Vincent (1997) (“M&V”) *except* that in M&V, the sales mechanism is a sealed-bid second-price auction (“SPA”) as opposed to an APA.

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<sup>1</sup>Cassady (1967), pages 57, 105, and 113.

This distinction is interesting from an auction-theory standpoint due to the well known equivalence between the APA and the SPA in static settings.<sup>2</sup> Formally, if buyers' valuations are independent of one another, then the amount a buyer would choose to bid in a SPA is the same price at which he would drop out in an APA. As a consequence, the seller's optimal reserve price in a SPA is also her optimal starting price in an APA and the two auction formats are revenue equivalent. Within the dynamic framework considered here, the two formats are no longer strategically equivalent. They are, as it turns out, still revenue equivalent.

To understand how the strategic equivalence between auction formats fails, we begin by considering the model in M&V. M&V wed the literature on optimal auctions to the literature on sequential bargaining with one-sided incomplete information.<sup>3</sup> As in the optimal auctions literature, a seller of a single item faces a set of buyers, each with unit demands. Buyers have private information over their own valuations, referred to as their *type*, while the valuations of others are unknown, assumed to be independent and identically distributed. As in the sequential bargaining literature, the horizon is infinite and buyers are long-lived. In each period, the seller may run a SPA with an announced reserve price. The seller cannot commit to a predetermined reserve-price path, nor can she commit to keep the item off the market. Therefore, if the auction in a given period fails to produce a sale, she relists in the following period, and continues to run successive auctions until a sale is eventually transacted.

The equilibrium of the game is characterized by a marginal type in each period such that all buyers whose valuations exceed the marginal type, or *screening level*, bid their valuations. In this way, if the item fails to sell, the seller infers that all types are below the screening level and reduces the reserve price in the following period accordingly.

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<sup>2</sup>See Vickrey (1961).

<sup>3</sup>See Meyerson (1981) and Riley and Samuelson (1981) on optimal auctions. Gul, Sonnenschein, and Wilson (1986), Fudenberg, Levine, and Tirole (1985), and Sobel and Takahashi (1983) model sequential bargaining with one-sided incomplete information.

The screening level is determined to be the type indifferent between taking the item off the market in the current period and waiting one additional period—where the reserve will be reduced—before bidding.

Now consider the APA version of the sequential auction game. In the typical English auction, an auctioneer announces the starting price and calls for bids. Say the starting price was determined to be \$100, the auction might very well begin with the auctioneer announcing, “One hundred dollars. Do I hear one hundred?” If a bid is proffered, the price is raised by some increment and the auctioneer calls out for further bids. In the context of our example, the auctioneer might say, “We have one hundred dollars. Do I hear one twenty?” The bidding continues in this manner until the auctioneer reaches a price at which no buyers wish to continue bidding. But, if no bid is proffered at the starting price, we assume, as in M&V, that the item is relisted at some point in the future which we model as the subsequent period. The auction described by Cassady is a special case of this setting, one in which the time between periods is infinitesimally short. In that case, the auctioneer, upon failing to find any takers at \$100, immediately retreats to \$80, say, and announces, “Eighty dollars. Do I hear eighty?”

Analogous to the sequential SPA game, the sequential APA game is characterized by a marginal type in each period indifferent as to whether or not to bid at the starting price. Consider then a buyer in the APA game who had planned to wait for a future period before placing a bid. Once a bid is placed (by some other buyer) at the starting price, the value of waiting vanishes as the item will be gone before the next period arrives. Under these circumstances, such a buyer can do no better than to bid up to his valuation in the current period. Such retaliatory bidding raises the price paid by the initial bidder above what he would have paid were the sales mechanism a SPA. As a result, the decision to bid at the starting price is more costly in the APA game, all else equal, knowing that doing so invites bids from those who would have otherwise abstained. Thus, the marginal bidder type in the SPA game prefers not to bid at the

starting price in the APA game. This logic assumes that the sequence of starting prices and screening levels in the APA game are equivalent to the sequence of reserve prices and screening levels in the SPA game in all periods but the one in question, so that the only difference between the APA game and SPA games is the retaliatory bidding.

The seller's strategy consists of choosing a starting price in each period giving rise to the desired screening level. In the APA version of the game, the screening level determines the set of buyers willing to bid at the starting price. These are the buyers who find it profitable to bid without knowing whether others intend to do so. By the logic of the previous paragraph, the retaliatory bidding by those who bid only after the starting price is met makes the decision to place the initial bid more costly. As such, the seller must reduce the starting price from what it would have been in a SPA if she intends to induce bids by the same types.

I show that the seller in the APA game chooses a sequence of starting prices that induces the identical sequence of screening levels as in the SPA game. This result gives rise to the revenue equivalence of the two auction formats. However, the sequence of starting prices in the APA game is shown to be no greater, term by term, than the sequence of reserve prices in the SPA game. In any game lasting longer than one period, the starting price in a given period is strictly less than the analogous reserve price in all periods but the last.

The resulting difference between a starting price in an APA and the analogous reserve price in a SPA has interesting implications regarding the Coase conjecture. Coase (1972) considered a durable good monopolist who could not commit to restricting sales in a given period. He famously conjectured that as the time between periods becomes very small, the price in the initial period should converge to the seller's marginal cost. Gul, Sonnenschein, and Wilson (1986) went on to prove the Coase conjecture and in doing so pointed out a mathematical equivalence between durable good monopoly and

sequential bargaining with one-sided incomplete information models.<sup>4</sup> Restated in the terminology of the sequential bargaining literature, the Coase conjecture states that as the time between periods becomes very small, the seller's initial offer converges to the minimum valuation type.<sup>5</sup>

M&V proved a corollary of the Coase conjecture (Theorem 3) by showing that as the discount factor approaches unity, the seller's revenue approaches that of a seller who auctions the item without a reserve price in the initial period. I refer to this result as a corollary as Coase's actual prediction was about prices, with the effect on revenue a natural consequence.<sup>6</sup> In fact, M&V show (Corollary 3) that in the sequential SPA game, the seller's initial reserve price is bounded away from the minimum type. But in the sequential APA game, as the discount factor approaches unity, the seller's initial starting price converges to the minimum valuation type, thereby extending the Coase conjecture to the sequential auction setting.

Returning to Cassady's account of opening-bid fishing, Cassady appears to be describing the sequential APA process with a very short time to relisting. This makes the sequential APA model appropriate for studying such behavior and the resulting price dynamics. The price dynamic in which an auction started at  $S$ , say, fails to sell while a subsequent auction started at a lower price ends with a price in excess of  $S$ , while seemingly paradoxical, is in fact a natural consequence of the sequential process.

The explanation I provide follows directly from the description given by Cassady. That is, if no bids are received at the initial starting price, the starting price is reduced until a bid is received. Anticipating such behavior, rational consumers may wish to hold off bidding in the current auction and instead wait for the subsequent auction where the

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<sup>4</sup>See Waldman (2003) for a survey of durable goods theory.

<sup>5</sup>In fact, the result states that the seller's initial offer converges to the greater of the seller's valuation and the minimum buyer valuation type. However, when the seller's valuation exceeds the minimum buyer valuation type, the "no gap" case, there may be other equilibria that do not give rise to this convergence result.

<sup>6</sup>See Coase (1972) pg. 143 for his famous "twinkling of an eye" argument.

item can be obtained at a lower price. For instance, a buyer with a valuation of  $S$ , say, gains nothing by winning an auction started at  $S$ . But should the item go unsold in the initial auction, he earns positive expected surplus in the following period when the starting price is reduced. Thus, buyers whose valuations just exceed  $S$  hold off bidding in the auction started at  $S$  but bid up to their valuations when the item is subsequently relisted with a lower starting price.

The preceding argument shows that the price dynamics described by Cassady is a possible outcome. I propose to go further and show that it is in fact the likely outcome. Toward that end, I ask whether an auction started at some price below  $S$  is more likely to end above  $S$ , conditional upon having failed in a previous auction with a starting price of  $S$ , than the initial auction with a starting price of  $S$ . Since this condition is stronger than the mere possibility of such an outcome, I distinguish between *strong* and *weak start-price end-price gaps*. For a given  $S$ , the weak start-price end-price gap necessarily holds while the strong start-price end-price gap holds only under certain parameterizations of the model. However, I show that in equilibrium the starting prices chosen in the sequential APA game are such that the strong start-price end-price gap results when the discount factor is close to unity i.e. when the time between auctions is quite short. This result does not extend to the sequential SPA game.

The paper is organized as follows. Section 2 describes the basic sequential SPA model considered by M&V then extends it to APAs. Section 3 characterizes the equilibrium of the sequential APA game and proves the Coase Conjecture. For purposes of comparison, Section 4 presents the solution to the analogous game in which the seller can commit to a predetermined starting price path. Section 5 derives some economic implications of the no-commitment model, focusing primarily on the start-price end-price gap. Section 6 concludes.



## 2 Sequential Auction Model

The following describes the sequential APA model, which is an extension of the model analyzed by M&V.

Consider an auction marketplace consisting of a single seller and  $n \geq 1$  potential buyers indexed  $i = 1, 2, \dots, n$ . The seller has one unit of a particular item to sell. Buyers are risk neutral, have unit demands, and differ only in their valuation of the item. Each buyer's valuation, denoted  $v$ , is private information. Valuations are assumed to be independent and identically distributed according to  $F$ , a continuous distribution with density  $f$ , bounded between zero and infinity, over the support  $[\underline{v}, \bar{v}]$ , where  $\underline{v} > 0$ . The seller has no value for the item, is risk neutral, and offers the item for sale using an optimal auction.<sup>7</sup> Assume the seller's valuation, the distribution of buyer valuations  $F$ , and  $n$  are common knowledge.

The optimal auction literature maintains that under the given assumptions over buyer valuations, all the of the well-known auction formats are revenue-equivalent in a one-period setting.<sup>8</sup> M&V consider two of these formats, the second-price auction and the first-price auction ("FPA"), within the dynamic setting described below. In both of these formats, bids are submitted in sealed envelopes which are only opened after the bidding has concluded. In either format, the seller's strategy consists of an announced reserve price  $r_t$  in every period  $t$ . M&V show that there exists an equilibrium of the sequential FPA game that is revenue equivalent to the unique equilibrium of the SPA game and that has the seller choosing the identical sequence of reserve prices. The probability of sale in each period and the allocation of the item are also equivalent across formats.

In the current setting, we consider another one of the well-known auction formats, the English ascending-price auction. Analogous to a reserve price in the sealed bid

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<sup>7</sup>The seller's valuation need not be zero. However, it is important for the analysis that it be strictly less than  $\underline{v}$ . This is known as the "gap" case in the literature.

<sup>8</sup>See McAfee and McMillan (1987) or Milgrom (1989) for a survey on auction theory.

formats, the seller chooses a starting price  $s_t \in [0, \infty)$ , which is the price at which the bidding begins. Beginning with  $s_t$ , the auctioneer calls for bids. If a bid is placed, the price is raised by an infinitesimally small increment and bids are solicited from the remaining buyers. The price continues to rise as further bids are placed until a price is reached at which the bidding stops. The item then goes to the last buyer to have submitted a bid at the price of his last bid.<sup>9</sup> As such, a buyer pays the starting price only when he is the only potential buyer to have entered a bid.

To incorporate the sequential nature of the game, consider an infinite horizon with periods indexed  $t = 1, 2, \dots$ . In period 1, the seller runs an APA (SPA) with a starting (reserve) price of  $s_1$  ( $r_1$ ). Beginning with  $s_1$ , the auctioneer calls for bids. If a bid is placed, the auction proceeds as described previously. If no bids are placed at  $s_1$ , the seller may run an APA in period 2. If the item fails to sell in period 2, the game continues on to period 3 and so on. As the game advances from one period to the next, all buyers and the seller discount returns accrued in the following period by factor  $\delta \in [0, 1)$ , which is common knowledge.

M&V show that the seller's objective function need not be concave to insure a unique equilibrium. Such considerations complicate the analysis without adding to the economic substance and so I make the following concavity assumption.

**Assumption 1** For any  $v \in [\underline{v}, \bar{v}]$ ,  $2f(v) + vf'(v) > 0$ .

The model describes a sequence of APAs, where each period's auction gives a buyer the opportunity to take the item off the market. Should a buyer choose to bid, the auction necessarily results in a sale and the game ends at the conclusion of the bidding. In the literature on optimal auctions, it is commonly assumed that a seller can commit herself

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<sup>9</sup>Note that this model of the ascending-price auction differs from the "button push" auction of Milgrom and Weber (1982) commonly used to model English auctions. The difference is that in the current model, a buyer may enter the bidding after the bidding has started, which is more congruent with reality. Within the sequential setting, the outcome of the "button push" auction is equivalent to the SPA.

against future sales in the event the item goes unsold in the initial offering. Relaxing this assumption has the seller continue to relist as a sale to the lowest valuation buyer is always preferred to keeping the item. It is important to note that the current setup precludes the existence of *reputational equilibria* (Ausubel and Deneckere 1989), in which the seller reduces the starting price at an arbitrarily slow rate, given our assumption that  $\underline{v} > 0$ .

The sequential APA game is shown to have multiple equilibria. All of the equilibria are equivalent in revenue, in the probability of sale in each period, and in the allocation of the item. Using the appropriate equilibrium selection criterion, we settle on a *most reasonable* equilibrium of the APA game that is outcome equivalent to the SPA game. However, the sequence of starting prices in this equilibrium is strictly lower than the sequence of reserve prices in the sequential SPA game in all periods but the last.

### 3 Equilibrium of Sequential APA Game

The equilibrium concept is *perfect-Bayesian equilibrium* (“PBE”). I show that a continuum of equilibria in symmetric strategies exists. The equilibria differ in small ways so that the allocation of the item as well as the probability of sale and the seller’s expected revenue are equivalent. Any equilibrium is a history-contingent sequence of the seller’s starting prices  $s_t$ , buyers’ bidding decisions, and updated beliefs about the valuations of existing buyers satisfying the typical consistency conditions.

To state this formally, let  $H_\tau = \{s_1, s_2, \dots, s_\tau\}$  denote the history through period  $\tau$  of a game that has not ended prior to period  $\tau$ . Since a bid placed in any period  $t < \tau$  necessarily results in the game ending in that period,  $H_\tau$  consists only of the seller’s starting prices with the implicit assertion that no bids have been placed to that point. A strategy for the seller is a starting price  $s_\tau$ , optimal given her beliefs over buyer valuations. A strategy for each buyer consists of a decision of whether or not to bid

in a given auction and if so how high to bid. I restrict attention to monotonic bidding strategies, a requirement of an equilibrium in symmetric strategies.

The dynamic of the game is governed by a buyer's decision to bid at the starting price, that is when it is not known if other buyers intend to bid in that period. Until such a bid is placed, the history of the game consists of a sequence of starting prices that failed to induce bids. That all changes when a buyer starts the bidding in a given period, thus insuring that the item will sell and the game will end at the conclusion of the bidding. For this reason, I distinguish between *initial bidders*, those willing to bid at the starting price, and *interim bidders*, those who bid only after the bidding has begun. Note that there can be more than one initial bidder since the distinction is a counter-factual, determined by what the buyer *would do* if no other buyers had submitted a bid.

**Lemma 1** *The following characterize the equilibrium in any PBE:*

1. *An initial bidder, when not currently the high bidder, has a unique weakly dominant strategy calling for him to continue the bidding at any price up to his valuation.*
2. *In any period  $t$ , there exists a marginal type  $\beta_t$ , such that every buyer whose valuation exceeds  $\beta_t$ , bids at the starting price.*
3. *Regardless of the history, all buyer types bid at the starting price when the starting price is at or below  $\underline{v}$ .*
4. *There exists a period  $T < \infty$ , endogenously determined, such that the game ends in at most  $T$  periods.*

Part 1 of the lemma extends Vickrey's result to the sequential SPA. This strategy is the unique symmetric strategy amongst initial bidders and so we assume initial bidders follow this strategy in what follows. The strategy of interim bidders is as yet undetermined.

Part 2 of the lemma is the *successive skimming* property, establishing the *Coasian* nature of the game. Analogous to the dual literatures on sequential bargaining and durable good monopoly, a buyer bids at a given starting price only if the starting price,  $s_t$ , is below some endogenously determined maximum. The type  $\beta_t \geq s_t$  is the lowest valuation type for which  $s_t$  is below such a maximum. Since any buyer whose valuation exceeds  $\beta_t$  necessarily bids in period  $t$ , it follows that if the item remains unsold after period  $t$ , it must be that all valuations are below  $\beta_t$ . This makes for a simple updating rule in which  $\beta_t$  becomes the highest type in period  $t + 1$ , denoted  $u_{t+1}$ . In what follows, we refer to  $u_\tau$  as the *state* in period  $\tau$ . Further, this result proves that initial bidders have higher valuations than interim bidders. Since the screening level,  $\beta_t$ , is decreasing over time, any interim bidder in period  $t$  must not have had a valuation above  $\beta_t$ , otherwise he would have been an initial bidder.

It should also be noted that the term *screening level* takes on a new meaning within the sequential APA game. In the sequential SPA and FPA games, the screening level  $\beta_t$  serves as both the minimum type to bid in period  $t$  as well as the value such that  $F(\beta_t)^n$  is the probability that the item remains unsold following the period  $t$  auction. Only the latter extends to the sequential APA game.

The first two parts of the lemma combine to indicate that the allocation of the item amongst buyers will be unaffected by the strategies of interim bidders since the item can only be obtained by the buyer with the highest valuation. What the bidding of interim bidders can do is raise the price above what it would have been in a SPA, conditional upon a sale taking place in a given period. To see this, consider the bidding decision of an interim buyer after the bidding has begun. The fact that he was not an initial bidder indicates to him that he will ultimately lose the bidding and so has no reason to bid. On the other hand, he has no reason not to bid as doing so is costless. This leads to a mixed strategy whereby an interim buyer mixes between bidding and not bidding at any possible price below his valuation. After proving an equivalence amongst all mixed

strategy equilibria, I justify a selection criteria that has all such buyers bid up to their respective valuations.

The result that the screening level eventually reaches  $\underline{v}$  (result 4), whereby the game necessarily ends (result 3), implies that the equilibrium can be derived via backward induction. The number of periods required for the starting price to reach  $\underline{v}$  is determined endogenously, thus complicating the analysis as the number of periods to be inducted upon has yet to be determined. The approach taken in the literature (Fudenberg, Levine, and Tirole 1985; McAfee and Vincent 1997) has been to define a game that is arbitrarily constrained to end in a predetermined number of periods with a starting price of (or no greater than)  $\underline{v}$ . Analysis of the arbitrarily constrained game then provides insight into how the number of periods is endogenously determined in the equilibrium of the unconstrained game. In particular, we show that there exists a sequence of cutoff values  $\{z_t\}_{t=1}^T$ , such that if the seller believes the highest buyer valuation to be in the interval  $(z_{t-1}, z_t]$ , she chooses a starting price that has the game end in  $t - 1$  additional periods. The interested reader can refer to Appendix A.2 for the formal analysis.

For a given  $T$  so defined, the equilibrium is characterized by the following sequences,

$$\{\beta_t\}_{t=0}^T, \{\sigma_t\}_{t=0}^T, \{\Gamma_t\}_{t=0}^T, \{g_t\}_{t=0}^T$$

defined as follows.

Let  $\sigma_t(x)$  denote the starting price that induces a screening level of  $x$  in period  $t$ . Since the type- $x$  buyer wins the auction only upon being the lone initial bidder,  $\sigma_t(x)$  satisfies

$$[x - \rho_t(\sigma_t(x), x)] F_{Y_1}(x) = \delta \left( [\beta_{t+1} - \rho_{t+1}] F_{Y_1}(\beta_{t+1}) + \int_{\beta_{t+1}}^x F_{Y_1}(Y_1) dY_1 \right). \quad (1)$$

The left-hand side of the expression indicates a type- $x$  buyer's expected surplus from

bidding at starting price  $\sigma_t(x)$ , when he is the lowest type to be an initial bidder. His expected payment under the circumstance is  $\rho_t(\sigma_t(x), x)$ . In a SPA, absent interim bidders, his payment would be  $\sigma_t$ . However, since a bid at the starting price induces bids from those who otherwise would have waited for a subsequent period prior to bidding, his payment goes up. The term  $\rho_t(\sigma_t(x), x)$  denotes the expected maximum bid of all interim bidders, given that such a bid is no greater than  $x$ . Since all interim bidders are indifferent between not bidding and bidding at any price up to their valuations, the determination of  $\rho$  is completely arbitrary. We assume in what follows that the function  $\rho(\cdot, \cdot)$ , strictly increasing in both arguments, is known to all players and that each buyer, when contemplating the decision of whether to bid at the start, uses the same calculation of  $\rho$ .<sup>10</sup>

The expression  $F_{Y_1}(x)$  denotes the probability that buyer type  $x$  is the lone initial bidder. Using conventional notation, let  $Y_1$  denote the maximum of  $n - 1$  other buyer valuations. Since the  $v_i$  are independent,  $F_{Y_1}(x) \equiv F(x)^{n-1}$ .

The right-hand side of equation (1) gives a type- $x$  buyer's expected continuation surplus, given a starting price of  $\sigma_{t+1}$  and a screening level of  $\beta_{t+1}$  in the period to follow.<sup>11</sup> In the following period, since  $x \geq \beta_{t+1}$ , he receives an amount given by the first term, where  $\rho_{t+1} \equiv \rho(\sigma_{t+1}, \beta_{t+1})$ , in the event that he is the lone initial buyer and an amount given by the second term when he is bidding against at least one other initial bidder.

Let  $g_t(u, x)$  denote the seller's revenue from period  $t$  onward, in state  $u$ , when choosing a starting price that induces a screening level of  $x$ . We have that

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<sup>10</sup>The assumption that  $\rho$  is increasing in both arguments is satisfied in the SPA game (see M&V) and is later shown to be satisfied in the refined equilibrium of the APA game. The assumption that all buyers believe  $\rho$  to be the expected payment to the marginal bidder type is no more restrictive than assuming common beliefs.

<sup>11</sup>More precisely, the right-hand side of (1) gives the expected surplus to the indifferent type *unconditional* on the following period being reached. As such, the expression can be read as the expected surplus, conditional on the following period being reached, multiplied by the probability that the following period is reached,  $F_{Y_1}(x)$ . Since the conditional expectation has as the denominator  $F_{Y_1}(x)$ , the  $F_{Y_1}(x)$  terms cancel.

$$g_t(u, x) = n\rho(\sigma_t(x), x) F_{Y_1}(x) [F(u) - F(x)] + n \int_x^u \int_x^{X_1} Y_1 dF_{Y_1} f(X_1) dX_1 + \delta \Gamma_{t+1}(x).$$

The first term represents the seller's revenue in the immediate period from having only one initial buyer and the second from having at least two. The third term represents her maximum discounted return from the following period on, at state  $x$ , in the event the current auction fails to produce a sale.

The seller's problem in period  $t$  can be thought of as choosing  $x$  to maximize the discounted sum of expected revenue from the current period on. In that case, we have that

$$\Gamma_t(u) = \max_{x \leq u} g_t(u, x).$$

The seller's optimal screening level in period  $t$ ,  $\beta_t$ , maximizes  $g_t(u, x)$ . Formally,

$$\beta_t = \arg \max_{x \leq u} \{g_t(u, x)\}.$$

The equilibrium is unique for a given function  $\rho$ . As mentioned earlier, the determination of  $\rho$  is completely arbitrary. As such any function that outputs a value over a continuum from the starting price to the maximum of  $Y_1 < \beta_t$  can be part of an equilibrium. Thus, we have a continuum of equilibria which are characterized by the following proposition.

**Proposition 1** *A PBE of the sequential APA game consists of a sequence of screening levels  $\{\beta_t\}_{t=1}^T$  and corresponding starting prices  $\{s_t\}_{t=1}^T$  such that:*



1. In any period  $t$ ,  $\beta_t = \beta(u_t)$  is chosen to satisfy

$$\beta_t f(\beta_t) + F(\beta_t) - F(u_t) \leq 0,$$

with a strict equality when  $\beta_t > \underline{v}$ . The choice of  $\beta_t$  depends only on  $u_t$  and the density  $f$ , independent of  $\delta$ ,  $n$ , and  $\rho$ .

2. The period- $t$  starting price  $s_t = \sigma(\beta_t)$  satisfies equation (1) and induces a screening level of  $\beta_t$  for a given  $\rho$ .

3. The allocation of the item, the seller's revenue, and the probability of sale in a given period are independent of  $\rho$ .

The result that the seller's revenue is independent of  $\rho$  would seem unintuitive since a higher value of  $\rho$  translates to a higher expected payment for a buyer in a given period, holding constant the seller's starting price. However, the seller takes  $\rho$  into account when selecting the starting price, ultimately selecting a starting price such that the screening level is invariant to  $\rho$ . Since  $\rho(\sigma(\beta_t), \beta_t)$  equal to  $\sigma(\beta_t)$  is a special case of our model, the sequential SPA mechanism is revenue-equivalent to any equilibrium of the sequential APA game.

Analysis of the stationary problem defined by Lemma 1 shows that the seller's problem in a given state is solved by a choice of  $\beta_t$  that depends only on the present state and on the distribution of valuations. It then follows from the buyers' sequential rationality condition, equation (1), that there is a unique  $\rho_t$ —a function of  $\delta$  and  $n$ —in each period that makes a type  $\beta_t$  buyer indifferent between bidding at the starting price and waiting for the following period. Having determined  $\rho_t$ , the starting price is some value of  $s_t$  giving rise to  $\rho_t$ .

The indeterminacy of the equilibrium starting-price path can be broken with the appropriate equilibrium selection criteria. In fact, we reduce the multiplicity of equilibria

to a single point by considering the appropriately perturbed version of the game. The perturbed game we consider is one in which, in a given auction, every possible move is played by each buyer with positive probability.<sup>12</sup> For any price in  $[s_t, \bar{v}]$  called out by the auctioneer, each buyer bids with some probability and does not bid with positive probability. The limits of equilibria of such perturbed games as the tremble probabilities go to zero are *extensive-form trembling-hand-perfect equilibria* (“ETE”).<sup>13</sup>

**Lemma 2** *There exists a unique ETE of the sequential APA game. In the ETE, each interim buyer bids up to his valuation with probability 1.*

Since the equilibrium characterized by Lemma 2 is the only one to meet our selection criteria, we assume such behavior in our further discussion of the APA. Therefore, in the APA, the expected payment made by a lone initial bidder is

$$\begin{aligned}\rho(s_t, \beta_t) &= E[\max\{s_t, Y_1\} | Y_1 < \beta_t] \\ &= \beta_t - \frac{\int_{s_t}^{\beta_t} F_{Y_1}(Y_1) dY_1}{F_{Y_1}(\beta_t)}.\end{aligned}\tag{2}$$

Perhaps the most profound difference between the sequential SPA model of M&V and the APA considered herein, is in the derivation of the Coase conjecture. It was mentioned in the introduction, that in the SPA, even as  $\delta \rightarrow 1$ , the seller’s initial starting price is bounded above  $\underline{v}$ . In the APA, when buyers’ bidding is consistent with Lemma 2, the starting price required to induce a given screening level is below that of a SPA and is not bounded away from  $\underline{v}$ .

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<sup>12</sup>Trembles by the seller are uninteresting, since the starting price is known to all buyers before the auction begins.

<sup>13</sup>The ETE is an extension, due to Selten (1983), of the *trembling-hand-perfect* equilibrium concept of Selten (1975). A trembling-hand perfect equilibrium is one that takes the possibility of off-the-equilibrium play into account by assuming that the players, through a tremble, may choose unintended strategies, albeit with negligible probability. When extending this concept to extensive-form games, the modeler may choose to interpret a tremble as a mistake in a player’s choice of action at a particular information set or as a mistake in a player’s entire strategy choice. The ETE concept employs the former interpretation.

**Proposition 2** *In the ETE of the sequential APA game, for every  $\varepsilon > 0$ , there exists a  $\bar{\delta} < 1$  such that for all  $\delta \geq \bar{\delta}$ , and for any initial screening level,  $\beta_1 = \beta(\bar{v}) \in [\underline{v}, \bar{v}]$ , the seller's initial starting price,  $\sigma(\beta_1)$ , is less than  $\underline{v} + \varepsilon$ .*

It is straightforward to see why the APA emits a Coase conjecture while the SPA does not. In either model, the seller's optimal initial screening level,  $\beta_1$ , is equal to the optimal reserve in a one-shot auction, independent of  $\delta$ . The seller is then charged with the task of choosing a starting price to induce a screening level of  $\beta_1$ . In the limit as  $\delta$  approaches unity, this gives rise to the following equality:

$$\lim_{\delta \rightarrow 1} (\beta_1 - \rho_1) F_{Y_1}(\beta_1) = \int_{\underline{v}}^{\beta_1} F_{Y_1}(Y_1) dY_1,$$

where  $\rho_1 = \rho(\sigma(\beta_1), \beta_1)$ . In the ETE of the APA,  $\rho_1$  is given by (2). Substituting this value of  $\rho_1$  into the above yields

$$\lim_{\delta \rightarrow 1} \int_{\sigma(\beta_1)}^{\beta_1} F_{Y_1}(Y_1) dY_1 = \int_{\underline{v}}^{\beta_1} F_{Y_1}(Y_1) dY_1.$$

The only way for this equality to hold is for  $\sigma(\beta_1)$  to be equal to  $\underline{v}$  in the limit. Conversely, in the SPA  $\rho_1 = \sigma(\beta_1)$ , so the limiting equality becomes

$$\lim_{\delta \rightarrow 1} [\beta_1 - \sigma(\beta_1)] F_{Y_1}(\beta_1) = \int_{\underline{v}}^{\beta_1} F_{Y_1}(Y_1) dY_1.$$

It is evident that for this equality to hold in the limit,  $\sigma(\beta_1)$  must strictly exceed  $\underline{v}$  for any  $\beta_1 > \underline{v}$ .<sup>14</sup>

The intuition is that the seller must guarantee the type- $\beta_1$  buyer a reservation level

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<sup>14</sup>The right-hand side of the equality is the area under the  $F_{Y_1}$  curve over the domain  $[\underline{v}, \beta_1]$ . The left-hand side of the equality is a rectangle with height  $F_{Y_1}(\beta_1)$  and base  $\beta_1 - \sigma(\beta_1)$ . Since  $F_{Y_1}$  is weakly increasing, if  $\sigma(\beta_1)$  were to be equal to  $\underline{v}$ , then the area under the curve would lie entirely inside the rectangle so the equality fails. Thus, the value of  $\sigma(\beta_1)$  must be sufficiently above  $\underline{v}$  to bring about an equality.

surplus to induce him to bid in the initial auction. For  $\delta$  close to unity, the buyer is sufficiently patient that his reservation surplus is close to the surplus he would otherwise have gotten in the last period of the game, a one-shot auction with a starting price of  $\underline{v}$ . In the SPA, the seller screens out types in the interval  $[s_1, \beta_1)$  with a binding reserve price of  $s_1$ . Thus the type- $\beta_1$  buyer pays  $s_1$  for realizations of  $Y_1$  in  $[s_1, \beta_1)$ . By choosing a sufficiently small  $s_1$ , the seller can make bidding in the initial period profitable for the type- $\beta_1$ . But in the APA, the seller cannot screen out buyer types in the interval between the starting price and  $\beta_1$  since interim bidders are free to enter the bidding after the opening bid. Thus the type- $\beta_1$  buyer pays  $Y_1 > s_1$  for realizations of  $Y_1$  in  $[s_1, \beta_1)$ . Under those circumstances, the surplus earned by the type- $\beta_1$  buyer is equal to that of a one-shot auction with starting price  $s_1$ . A buyer comparing the surplus from a one-shot auction with starting price  $s_1$  to that of a one-shot auction with starting price  $\underline{v}$  will choose the former only if  $s_1 = \underline{v}$ . Thus the only way the seller can guarantee the type- $\beta_1$  buyer his reservation surplus is by running an auction with a starting price close to  $\underline{v}$ , where  $\underline{v}$  is the starting price in the last period of the game.

## 4 Commitment Solution

As a means of comparison, consider what happens when a seller can commit to a given starting price or more generally to a sequence of starting prices. The purpose behind this analysis is to show that a seller with commitment power limits sales to a take-it-or-leave-it offer so that a seller would not fish for an opening bid.

**Definition 1** *A commitment equilibrium is a sequence of  $\hat{T}$  starting price offers  $s_1^*, \dots, s_{\hat{T}}^*$  that maximize the seller's expected revenue, assuming buyers make their bidding decisions to maximize expected surplus while taking  $\{s_t^*\}_{t=1}^{\hat{T}}$  as given.*

The commitment solution requires the seller to choose  $\hat{T}$  optimally along with the sequence of starting prices. It would seem straightforward that a seller would want to allow  $\hat{T} > 1$  and commit to a declining reserve-price plan. This way, she can hope to capture a sale in a future period should the item fail to sell in the initial period. However, the declining path creates an incentive, however small, for buyers whose valuations exceed the initial starting price to delay bidding. This incentive reduces bidding and consequently expected revenue in the initial period. As in the literature on sequential bargaining (Sobel and Takahashi 1983), when buyers are at least as patient as the seller, the cost to the seller in terms of lost revenue in the initial period is at least as great as the future stream of revenue from following a declining path.

**Proposition 3** *In the commitment equilibrium of the sequential APA and SPA games, the seller makes a take-it-or-leave-it offer, which is an offer to participate in an auction with starting price  $s^*$ , and the game takes place over a single period. The optimal starting price  $s^*$  maximizes the seller's revenue in a one-shot auction, satisfying:*

$$s^* f(s^*) + F(s^*) - 1 = 0.$$

*Taking the seller's strategy as given, buyers have the following (weakly) dominant strategy: at any price  $p \geq s$ , bid if  $p \leq v$ ; do not bid otherwise.*

That the seller limits sales to a single period extends the analogous result of Sobel and Takahashi (1983) from sequential bargaining to sequential auctions. Similar results hold in the literature on intertemporal price discrimination (Stokey 1979) and with respect to discrimination along the dimension of quality (Johnson and Myatt 2003).

Two things become evident from the equilibrium characterization. First, since sales are limited to a single period, the strategic equivalence between the APA and the SPA from static settings extends to the current setting. Second, the seller (or an auctioneer

whose incentives are aligned with those of the seller) does not fish for an opening bid. By limiting sales to a single period, the seller mimics the optimal auction prescribed by Meyerson (1981) and Riley and Samuelson (1981).

## 5 Price Dynamics

The introduction described the (weak) start-price end-price gap, which appears to result from the sale of a single item via a sequence of ascending-price auctions. Cassady explained that after failing to generate any bids in the initial auction, upon lowering the starting price, the seller is able to generate bidding that escalates beyond the amount of the initial starting price. The weak start-price end-price gap states that this is a possible outcome. The strong start-price end-price gap is a probabilistic statement that the subsequent auction is actually *more likely* to end with a price at least as high as the initial starting price than is the initial auction. In this section, I show how both versions of the start-price end-price gap can result in the equilibrium of the sequential APA game. This result is compared to the sequential SPA game in which only the weak version results.

Before proceeding, it is useful to formalize the concept of a start-price end-price gap. The weak version looks at the possibility of an auction ending above some price  $S$  after an earlier auction with a starting price of  $S$  failed to result in a sale.

**Definition 2** *Define the random variable  $p_t$  as the end price in the period- $t$  auction, assuming equilibrium behavior. Let  $\eta_t$  denote an indicator function taking a value of 1 if the period- $t$  auction fails to induce a sale, 0 if it does. We say that a weak start-price end-price gap results if there exist values  $t$  and  $t + \tau$ ,  $\tau \geq 1$ , such that for  $S > \underline{v}$ ,*

$$P \{ p_{t+\tau} \geq S | S = s_t, \eta_{t+\tau-1} = 1 \} > 0.$$

The definition of a strong start-price end-price gap compares the probability of sale

in some period- $t$  auction, with a starting price of  $S$ , against the probability that some future period's auction ends with a price of at least  $S$ , conditional upon that period being reached.

**Definition 3** *We say that a strong start-price end-price gap results if there exist values  $t$  and  $t + \tau$ , such that for  $S > \underline{v}$ ,*

$$P \{p_{t+\tau} \geq S | S = s_t, \eta_{t+\tau-1} = 1\} > P \{p_t \geq S | S = s_t\}.$$

It is clear from Definitions 2 and 3, that the strong start-price end-price gap implies the weak, but the converse does not hold. Implicit in the definitions of 2 and 3 is that  $S$ , the period- $t$  starting price, may be chosen exogenously, as a parameter in the analysis. The behavior of bidders in period  $t$  and the behavior of all players in the  $\tau$  subsequent periods is assumed to be in equilibrium.

## 5.1 Weak Start-Price End-Price Gap

In contrast to the commitment equilibrium, the seller fishes for an opening bid in the no-commitment equilibrium. Should the previous period's auction fail to induce a sale, the starting price is reduced and the seller again calls for bids. Buyers condition their behavior on the assumption that the seller will behave as such. As a consequence, buyers must be guaranteed a certain reservation level surplus to be induced to bid in a given period. This results in a screening level that strictly exceeds the starting price in any period except the terminal period. As such, the fact that a previous auction failed to sell with a starting price of  $S$  does not imply that the highest buyer valuation is below  $S$ . What it means is that the highest valuation is below the screening level in that auction. Since the screening level in an auction with a starting price of  $S$  strictly exceeds  $S$ , it may still be possible for a subsequent auction to end in a price exceeding  $S$ . This logic

is formalized in what follows.

The equilibrium of the no-commitment game consists of a deterministic sequence of the seller's starting-price path  $\{s_t\}_{t=1}^T$  along with a corresponding sequence of screening levels  $\{\beta_t\}_{t=1}^T$  such that a buyer bids at the opening bid in period  $t < T$  if his valuation falls in the interval  $[\beta_t, \beta_{t-1})$ . Since a player with valuation  $\beta_t$  wins the period- $t$  auction only when  $Y_1 < \beta_t$  and thus pays  $\rho_t$ ,  $\beta_t$  must satisfy:

$$(\beta_t - \rho_t) F_{Y_1}(\beta_t) = \delta \Pi(\beta_t). \quad (3)$$

Equation (3) expresses indifference between the payoff from bidding in the current period and the discounted continuation payoff to a type- $\beta_t$  buyer were he to wait for the following period. Since the seller, in equilibrium, would never run an auction that did not induce any buyer types to bid,  $s_{t+1}$  will be such that all types greater than  $\beta_{t+1} < \beta_t$  are initial bidders. It follows that the continuation payoff for a type- $\beta_t$  buyer (multiplied by the probability that the period- $t$  state is reached) is

$$\Pi(\beta_t) = \begin{cases} (\beta_{t+1} - \rho_{t+1}) F_{Y_1}(\beta_{t+1}) + \int_{\beta_{t+1}}^{\beta_t} F_{Y_1}(Y_1) dY_1 & \text{if } t < T - 1 \\ \int_{\underline{v}}^{\beta_t} F_{Y_1}(Y_1) dY_1 & \text{if } t = T - 1 \end{cases}. \quad (4)$$

In order to apply the results of the following lemma to both the SPA and APA games, we write the expected payment by a lone initial bidder as

$$\rho(s_t, \beta_t; \alpha) \equiv (1 - \alpha) s_t + \alpha \left[ \beta_t - \frac{\int_{s_t}^{\beta_t} F_{Y_1}(Y_1) dY_1}{F_{Y_1}(\beta_t)} \right], \quad \alpha \in [0, 1].$$

In this way, the extreme cases of  $\alpha = 0$  and  $\alpha = 1$  denote the expected payment by the lone initial bidder in the SPA and APA respectively.

Equations (3) and (4) define a function  $\sigma(\beta_t; \alpha)$  such that the seller chooses a starting price of  $\sigma$  in order to induce a screening level of  $\beta_t$  when the expected payment



by the lone bidder is  $\rho(\sigma, \beta_t; \alpha)$ . In this section, our purpose is to examine a sequence of starting prices used by a seller and make an inference about the probability of sale or the probability of a given price being reached in a given period. For that reason, let  $b(u_t, s_t; n, \delta, \alpha) = \beta(u_t)$  denote the inverse of  $\sigma$ , representing the screening level as a function of the starting price, satisfying (3) and (4). The interpretation is that if a starting price of  $s_1$  is employed in the initial auction, the probability of sale is  $1 - F(b(\bar{v}, s_1; \cdot))^n$ .

It is important to note that equations (3) and (4) hold even when the current starting price,  $s_t$ , is off of the equilibrium path. Since there is nothing unknown about the seller's payoff, deviations from the equilibrium path are uninformative to buyers and can thus be interpreted as random errors. Assuming that the seller follows the equilibrium path from period  $t + 1$  onward,  $b(u_t, s_t; n, \delta, \alpha)$  is the marginal consumer type in period  $t$ . This type is indifferent between bidding in the current period and waiting for the following period where the seller optimally chooses  $s_{t+1}$  given that the highest possible type in period  $t + 1$  is  $b(u_t, s_t; n, \delta, \alpha)$ .<sup>15</sup>

Whether on or off the equilibrium path, the function  $b(\cdot)$  has some interesting economic properties that are laid out in the following lemma.

**Lemma 3** *Following any history of the game, the screening level in the current period,  $\beta_t = b(u_t, s_t; n, \delta, \alpha)$ , is:*

1. Increasing in  $s_t$ ;
2. Greater than  $s_t$ , strictly so for  $s_t > \underline{v}$ ;
3. Increasing in  $\alpha$ ;
4. Decreasing in  $n$ :  $\lim_{n \rightarrow \infty} b(u_t, s_t; n, \delta, \alpha) = s_t$ ;

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<sup>15</sup>Since the equilibrium for the game itself was solved assuming a generic initial prior  $\bar{v}$ , there exists an equilibrium of the subgame with initial state  $\beta_t$ .

5. Increasing in  $\delta$ :  $\lim_{\delta \rightarrow 0} b(u_t, s_t; n, \delta, \alpha) = s_t$ .

Property 1 says that the screening level is increasing in the starting price. This property is fairly trivial as it holds in the one-shot auction where the starting price and screening level are equal. More interestingly, property 2 says that there exists a gap between the starting price and the screening level for any starting price greater than  $\underline{v}$ . This is the result that allows for the price to exceed  $s_t$  in a subsequent auction after the auction started at  $s_t$  fails to result in a sale. To see this, suppose that after failing to sell the item at  $s_t$ , the seller reduces the starting price in such a way that  $\beta_{t+1}$  is the screening level in the following period. We know that  $\beta_{t+1} < \beta_t$ . Thus, if the two highest buyer valuations are between  $\max\{\beta_{t+1}, s_t\}$  and  $\beta_t$ , then the following period's auction ends with a price exceeding  $s_t$  while the auction started at  $s_t$  fails to generate a sale.

The preceding argument applies equally to the SPA as well as the APA indicating that the weak start-price end-price gap results in either format.

**Proposition 4** *In either the sequential APA or sequential SPA games in which  $T > 1$ , there exists a  $\bar{\tau} > 1$  such that for any  $\tau \leq \bar{\tau}$  and for any  $S > \underline{v}$ ,*

$$P\{p_{t+\tau} > S | S = s_t, \eta_{t+\tau-1} = 1\} > 0.$$

Proposition 4 says that if we consider any period  $t < T$  in which the period- $t$  auction fails to induce a sale, the following period's auction may end in a price of at least  $S$  with positive probability. In fact, any subsequent period's auction, say period  $t + \tau$ , may end in a price of at least  $S$  as long as  $\beta_{t+\tau-1}$  exceeds  $S$ . It follows then that  $\bar{\tau}$  is the highest value of  $\tau$  such that  $\beta_{t+\tau-1}$  exceeds  $S$ . In what follows, I show that along the equilibrium path, Cassady's outcome is in fact more likely to occur in the APA than in the SPA game. The result is due in large part to property 3 which shows that for a given starting price, the screening level is higher in the APA than in the SPA. Conversely, for

a given screening level, the equilibrium starting price in an APA is below that of the equilibrium reserve price in a SPA. The reduced equilibrium starting price in the APA, in essence, lowers the threshold that must be met for the strong start-price end-price gap to occur. Fixing the sequence of screening levels, the initial starting price in the APA game is more likely to be reached in some subsequent period than is the initial reserve in the SPA game. This logic is formalized in the following subsection.

Before proceeding, we note how the number of bidders and the common discount factor affect the equilibrium starting price path. Proposition 1 showed that the equilibrium sequence of screening levels is chosen independent of  $n$  and independent of  $\delta$ . Property 4 of Lemma 3 shows that in each period, the starting price required to induce the chosen screening level is higher with more buyers. Having more buyers in the auction increases competition for the item and reduces the reservation level surplus of a given buyer type. Thus the magnitude of the current period's surplus required to induce a given type to bid is reduced. As the number of buyers goes to infinity, the reservation level surplus goes to zero and so the starting price is chosen so as to give the indifferent bidder type zero surplus in the current period. In this limit, the starting price path of the APA game and the reserve price path of the SPA game each converge to the sequence of screening levels.

Property 5 of Lemma 3 shows that in each period, the higher is the discount factor the lower is the starting price required to induce the chosen screening level. As the discount factor goes to unity, Proposition 2 showed that the starting price converges to  $\underline{v}$ . Conversely, for a given screening level, the lower is the discount factor the higher is the starting price. A lower discount factor, interpreted as a greater time to the subsequent relisting, reduces the reservation level surplus for each buyer type. Thus, the surplus in the current period required to induce a given type to bid is reduced. As the discount factor goes to zero, so does the reservation level surplus. Thus, the starting price is set equal to the screening level, giving the indifferent buyer zero surplus. This result,

consistent with the commitment solution, applies equally to the APA and SPA games. In the commitment solution, since sales are limited to a single period, buyers behave as though  $\delta = 0$ . As such, the starting price in the APA game is equivalent to the reserve price in the SPA game.

## 5.2 Strong Start-Price End-Price Gap

Returning to the issue of the strong start-price end-price gap, we focus attention not on some arbitrary starting price,  $S$ , but rather on equilibrium starting prices  $\sigma(\beta_t; \alpha)$ ,  $\alpha \in \{0, 1\}$ . Definition 3 illustrates how the probability of reaching a price of  $\sigma(\beta_t; \alpha)$  differs across the two auction formats in question. Letting  $X_1$  and  $X_2$  denote the highest and second-highest buyer valuations respectively, we have that

$$P\{p_t \geq S | S = \sigma(\beta_t; \alpha)\} = P\{X_1 \geq \beta_t\}. \quad (5)$$

That is, the item sells in period  $t$  if the highest valuation exceeds the screening level, which is the same in both the APA and the SPA games. However, once the item fails to sell in period  $t$ , the probability that a subsequent auction reaches  $S$  differs across the two auction formats. We have that for any  $\tau \geq 1$ ,

$$\begin{aligned} & P\{p_{t+\tau} \geq S | S = \sigma(\beta_t; \alpha), \eta_{t+\tau-1} = 1\} \\ = & \begin{cases} P\{X_2 \geq S, X_1 \geq \max\{\beta_{t+\tau}, S\} | X_1 < \beta_{t+\tau-1}\} & \text{in APA} \\ P\{X_2 \geq \max\{\beta_{t+\tau}, S\} | X_1 < \beta_{t+\tau-1}\} & \text{in SPA} \end{cases}. \end{aligned} \quad (6)$$

In equation (6), the discrepancy between the SPA and APA stems from the fact that in the SPA, a buyer bids only when his valuation is above the screening level. But in the APA, once a bid is placed, indicating that *some* buyer's valuation is above the screening

level, then all buyers bid as long as the price is below their valuation. Thus for the APA to reach some threshold  $S$  requires that the highest valuation exceed  $\max\{\beta_{t+\tau}, S\}$ , while the second highest valuation exceeds  $S$ . But in the SPA, both the highest and second-highest valuations must exceed  $\max\{\beta_{t+\tau}, S\}$  for this to occur.

We are interested in exploring the conditions under which the strong start-price end-price gap results in the sequential APA and SPA games respectively. In doing so, it is important to note that the sequence of starting prices used in the APA game differ from the sequence of reserve prices used in the SPA game. In fact, they are lower, term by term, in all periods but the terminal period. In the terminal period, the starting price in the APA and the reserve price in the SPA are relevant only inasmuch as each induces a screening level of  $\underline{v}$ .<sup>16</sup> Suppose the period- $t$  starting price in the APA game is  $S$  and the period- $t$  reserve in the SPA game is  $R > S$ . The APA in period  $t + \tau$  ends with a price at least  $S$  if  $X_1 > \max\{\beta_{t+\tau}, S\}$  and  $X_2 > S$ . The SPA in period  $t + \tau$  ends with a price at least  $R$  if both  $X_1$  and  $X_2$  exceed  $\max\{\beta_{t+\tau}, R\} \geq \max\{\beta_{t+\tau}, S\}$ . We see then that a strong start-price end-price gap occurs under a larger set of parameterizations in the APA game because: 1. the APA induces bids from a larger set of bidder types; and 2. the fact that the period- $t$  starting price in the APA game is less than the period- $t$  reserve price in the SPA game, for  $t < T$ , makes a price of  $S$  easier to achieve than  $R$  in some subsequent period.

To simplify notation in what follows, define  $G^\alpha(S; t, \tau)$  such that,

$$G^\alpha(S; t, \tau) = P\{p_{t+\tau} \geq S | S = \sigma(\beta_t; \alpha), \eta_{t+\tau-1} = 1\} - P\{p_t \geq S | S = \sigma(\beta_t; \alpha)\},$$

$$\alpha \in \{0, 1\}$$

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<sup>16</sup>Result 1 of Proposition 1 says that the sequence of screening levels is independent of  $\rho$ . As such, both the APA and SPA games end in the same number of periods. By Property 3 of Lemma 1, the starting price or reserve price giving rise to a screening level of  $\underline{v}$  in the terminal period can be no greater than  $\underline{v}$ . Thus the period- $T$  starting price in the APA game needn't be lower than the period- $T$  reserve price so long as both are no greater than  $\underline{v}$ .

Following from Definition 3, we say that the strong start-price end-price gap results in the sequential APA game if  $G^1(S; t, \tau) > 0$ , and in the sequential SPA game if  $G^0(S; t, \tau) > 0$ . The expressions  $G^1$  and  $G^0$  provide us with a shorthand to express the relative probabilities contained in Definition 3.

The following result formalizes the argument that a start-price end-price gap necessarily occurs in a larger class of parameterizations for the APA game.

**Proposition 5** *For any period  $t < T$  such that  $\sigma(\beta_t; 1)$  and  $\sigma(\beta_t; 0)$  are the period- $t$  starting and reserve prices in the sequential APA and SPA games respectively, and for any  $\tau \leq T - t$ ,  $G^1(\sigma(\beta_t; 1); t, \tau) > G^0(\sigma(\beta_t; 0); t, \tau)$ .*

Though showing that it occurs in a larger parameter space in the APA game, Proposition 5 does not guarantee that a strong start-price end-price gap does, in fact, result. For this, we require  $G^1(S; t, \tau)$  to be positive. In either format, a strong start-price end-price gap becomes easier to achieve when the starting price in question,  $S$ , is lower. The Coase conjecture (Proposition 2) showed that as the time to relist becomes short, the sequence of starting prices in the APA game are such that they all lie within a neighborhood of the lowest valuation type. Thus, while the probability of a sale in the initial period is unaffected by  $\delta$ , the fact that the initial starting price is decreasing in  $\delta$  makes it more likely for that price to be reached in some subsequent period for  $\delta$  close to unity.

The following result shows that the strong start-price end-gap results in the sequential APA game when  $\tau = T - t$ .

**Proposition 6** *In the equilibrium of the sequential APA game, there exists some  $\tilde{\delta} < 1$  such that for any  $\delta \geq \tilde{\delta}$ , and for any  $t < T$ ,  $G^1(\sigma(\beta_t; 1); t, T - t) > 0$ , regardless of  $T$ , the number of periods required for the starting price to reach  $\underline{v}$ .*

The intuition for why the result holds for  $\tau = T - t$ , while not necessarily for  $\tau < T - t$  is as follows: Consider the probability that the auction in period  $t+1$  achieves a

price of  $S$  after the period- $t$  auction failed to induce a sale. For  $S$  to be reached in period  $t + 1$  requires the highest buyer valuation to be at least  $\max \{ \beta_{t+1}, S \}$ . For a large  $\beta_{t+1}$ , the probability of sale and hence the probability of  $S$  being reached may still be low. Recall that the sequence of screening levels is invariant to the discount factor. The same reasoning extends to all subsequent periods but the terminal period. In the terminal period, the screening level is equal to  $\underline{v}$  so the probability of sale is equal to one. Thus, if there exists a buyer with a valuation of at least  $S$ , and the item has remained unsold to that point, the period- $T$  auction will necessarily end with a price of at least  $S$ . And for  $S$  close to  $\underline{v}$ , the probability of there being a buyer with a valuation of at least  $S$  is close to unity.

By Proposition 6, we should not be surprised to see the price dynamic described by Cassady when the auctioneer lowers the start price in quick succession. This is because when doing so, the initial starting price becomes quite small. As such, upon failing to sell in the initial auction, and the  $T - 2$  subsequent auctions, the auction in period  $T$  will necessarily end in a price of at least  $\sigma(\beta_1; 1)$ . Proposition 6 shows that the period- $T$  auction is in fact more likely to end with a price of at least  $\sigma(\beta_1; 1)$ , conditional upon having reached period  $T$ , than is the initial auction. The reasons for this are: 1. the start-price end-price gap becomes easier to achieve when the initial starting price is quite small; and 2. when the time to relist is very short, the seller chooses the initial starting price sufficiently small so that the start-price end-price gap results. This result continues to hold when the period-1 starting price is replaced with period- $t$  starting price for any  $t < T$ . This result needn't hold in the sequential SPA game as each non-terminal period's reserve price, in general, is bounded above  $\underline{v}$  even for  $\delta$  close to unity.

The difference in outcomes across auction formats is illustrated in the following parametric example.

### 5.2.1 Parametric Example

For the purposes of this example, let  $n = 2$  and suppose that buyer valuations are drawn from a uniform  $[a, 1 + a]$  distribution for  $a \in (0, 1)$ .

The equilibrium of the sequential auction game is derived as follows. At any state  $u$ , the seller's choice of screening level satisfies the first-order condition

$$\beta f(\beta) + F(\beta) - F(u) \leq 0.$$

Since  $F(x) = x - a$  in this example, we have

$$\beta(u) = \begin{cases} u/2 & \text{if } u \geq 2a \\ a & \text{otherwise} \end{cases}. \quad (7)$$

At any state  $u$ , the seller cuts the demand curve in half, serving the top half, until  $u$  becomes sufficiently small so that  $u/2$  falls below  $a$ , whereby the screening level becomes  $a$  and all types are induced to bid.

The class of uniform  $[a, 1 + a]$  distributions makes for a useful example in that a game of any duration can be constructed by the appropriate choice of  $a$ . For  $a \geq 1$ , the seller ends the game in one period with a starting price of  $a$ . As  $a \rightarrow 0$ , the number of periods goes to infinity. Of course, when  $a = 0$ , result 4 of Lemma 1 no longer holds so the equilibrium characterized by equation (7) may no longer be unique. It follows from equation (7) that the game can be constructed to last  $T$  periods by choosing  $a \in [\frac{1}{2^T - 1}, \frac{1}{2^{T-1} - 1})$ .

We begin our examination of the strong start-price end-price gap with a two period example, so that  $a \in [1/3, 1)$ . The sequence of screening levels in these games is  $\{\frac{1+a}{2}, a\}$  and the period-2 starting/reserve price is  $a$  in either the APA or SPA game. In



the APA game, the period-1 starting price,  $s_1$ , satisfies

$$\int_{s_1}^{\frac{1+a}{2}} (y-a) dy = \delta \int_a^{\frac{1+a}{2}} (y-a) dy$$

so that

$$s_1 = a + \frac{1-a}{2} \sqrt{1-\delta}.$$

Notice that as  $\delta \rightarrow 1$ ,  $s_1$  goes to  $a$ , just as predicted by the Coase conjecture. The sequence  $\{s_1, a\}$  of starting prices gives rise to a strong start-price end-price gap if  $G^1(s_1; 1, 1) > 0$ , where,

$$G^1(s_1; 1, 1) = \frac{\int_{s_1}^{\beta_1} \int_{s_1}^{x_1} 2dx_2 dx_1}{\int_a^{\beta_1} 2(x-a) dx_1} - \left[ 1 - \left( \frac{1-a}{2} \right)^2 \right].$$

The result of Proposition 6 can be seen in this expression by setting  $s_1$  equal to  $a$ . The term in front of the minus sign goes to 1 as  $s_1 \rightarrow a$ , while the term after the minus sign remains bounded below 1 for any  $a < 1$ .

Figure 1.1 shows how  $G^1(s_1; 1, 1)$  varies with  $\delta$  for various values of  $a$ . Notice that for all values of  $a$ , there exists a  $\tilde{\delta}(a)$  such that  $G^1(s_1; 1, 1)$  is positive for all  $\delta \geq \tilde{\delta}(a)$ .

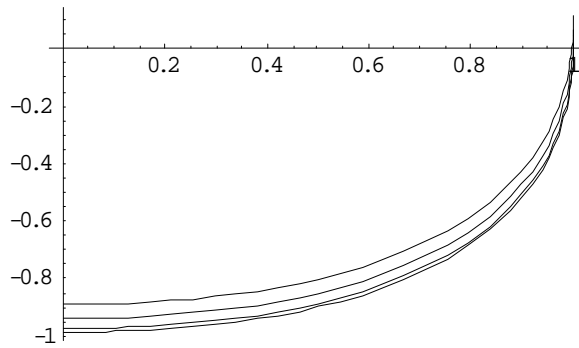


Figure 1.1:  $G^1$  as a function of  $\delta$  for  $a \in \{1/3, 1/2, 2/3, 3/4\}$ .

Now consider the SPA game within the same environment. As in the APA game, the sequence of screening levels is  $\{\frac{1+a}{2}, a\}$  and the period-2 reserve price is  $a$ . The only

difference is that the period-1 reserve,  $r_1$ , satisfies

$$\left(\frac{1+a}{2} - r_1\right) \left(\frac{1+a}{2} - a\right) = \delta \int_a^{\frac{1+a}{2}} (y-a) dy,$$

so that

$$r_1 = \frac{1+a}{2} - \frac{1-a}{4}\delta.$$

Notice that as  $\delta \rightarrow 1$ ,  $r_1$  goes to  $\frac{3a+1}{4}$ , *strictly* greater than  $a$ . The sequence  $\{r_1, a\}$  of reserve prices gives rise to a strong start-price end-price gap if  $G^0(r_1; 1, 1) > 0$ , where

$$G^0(r_1; 1, 1) = \frac{\int_{r_1}^{\beta_1} \int_{r_1}^{x_1} 2dx_2 dx_1}{\int_a^{\beta_1} 2(x-a) dx} - \left[1 - \left(\frac{1-a}{2}\right)^2\right].$$

Figure 1.2 shows how  $G^0(r_1; 1, 1)$  varies with  $\delta$  for various values of  $a$ . Notice that for all values of  $a$ ,  $G^0$  is negative for all values of  $\delta \in [0, 1)$ .

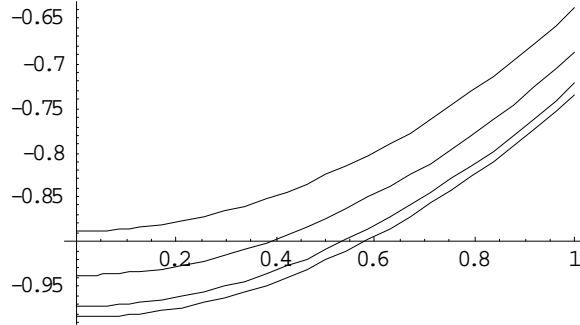


Figure 1.2:  $G^0$  as a function of  $\delta$  for  $a \in \{1/3, 1/4, 1/2, 3/4\}$ .

To show that the results derived from the two-period game extend to games of arbitrary length, we consider the game in which  $a \in [1/7, 1/3)$  so that the game lasts 3 periods. The three-period game consists of a sequence of screening levels  $\{\frac{1+a}{2}, \frac{1+a}{4}, a\}$ . The sequence of starting/reserve prices can be solved inductively beginning with the

period-3 starting price of  $a$ . In the SPA version of the game, the period-1 reserve is

$$r_1 = \frac{1+a}{2} - \frac{\delta}{16(1-a)} [(3-5a)(1+a) + \delta(1-3a)^2].$$

We then ask which auction is more likely to end with a price of at least  $r_1$ , the auction started at  $r_1$  or the auction started at  $a$  after having failed to sell at both  $r_1$  and  $r_2$ ? In order for the auction in period 3 to end in a price exceeding  $r_1$ , it must be the case that the screening level in period 2 is at least as high as  $r_1$ . If not, the fact that the period-2 auction failed to result in a sale implies that the highest buyer valuation is below  $r_1$ . Therefore, a necessary condition for the strong start-price end-price gap is  $\beta_2 > r_1$ . Since the limiting value of  $r_1$  as  $\delta \rightarrow \infty$  is  $(3a+1)/4$ , we have that  $r_1 \geq (3a+1)/4$ . We have that

$$\begin{aligned} \beta_2 &= (1+a)/4 \\ &< (3a+1)/4 \\ &\leq r_1, \end{aligned}$$

thus precluding the possibility of a strong start-price end-price gap within the SPA game.

This result can be extended to a game of any length. Since the seller uses the stationary strategy given by (7), we know that the screening level in the next-to-last period of a  $k$ -period game is

$$\beta_{k-1}^{(k)} = \frac{(1+a)}{2^{k-1}},$$

for  $a \in \left[ \frac{1}{2^{k-1}-1}, \frac{1}{2^{k-1}-1} \right)$ . Now consider the initial reserve price in a  $k$ -period game when  $\delta$  is arbitrarily close to 1. The limiting value of the initial reserve price,  $r_1$ , as  $\delta \rightarrow 1$  satisfies

$$\left( \frac{1+a}{2} - r_1 \right) \left( \frac{1+a}{2} - a \right) = \int_a^{\frac{1+a}{2}} (y-a) dy.$$

This is the indifference condition for the type  $\frac{1+a}{2}$  buyer in the initial period when the discount factor is set equal to 1. The expression simplifies to

$$r_1 = \frac{3a + 1}{4}.$$

We have that  $\beta_{k-1}^{(k)}$  is less than the limiting value of  $(3a + 1) / 4$  for any  $k \geq 3$ . Since the screening level in the next-to-last period falls below the initial reserve, the probability is zero that the SPA run in the terminal period can end with a price of at least  $r_1$ .

Contrast this to the  $k$ -period APA game. The sequence of screening levels is the same as in the SPA game. Therefore, the screening level in the next-to-last period is still  $(1 + a) / 2^{k-1}$ . In contrast to the SPA game, the limiting value of the initial starting price as  $\delta \rightarrow 1$  is the minimum valuation type,  $a$ . In the last period, all buyer types bid their valuations and so the price is determined by the second-highest valuation. The probability that the second-highest valuation exceeds  $a$ , conditional upon having failed to sell in all previous periods, is 1. Meanwhile, the probability that the price exceeds  $a$  in the initial period is equal to the probability that the highest valuation exceeds  $\beta_1^{(k)}$ . This event has probability less than 1, thus giving rise to a strong start-price end-price gap.

## 6 Conclusion

This paper began with the task of understanding the pricing dynamics that result when a seller (or auctioneer) goes fishing for the opening bid. I have developed a model of an English ascending-price auction in which the seller goes fishing for an opening bid when she cannot commit to a predetermined starting-price path. The weak start-price end-price gap described by Cassady is shown to be a natural consequence of the model and the strong start-price end-price gap results when the time between auctions

is sufficiently short.

Along the way, I distinguished the English ascending-price auction from the second-price auction within the sequential environment by reconsidering the manner in which the English auction is modeled. In much of the literature, the English auction is modeled using either the Milgrom and Weber (1982) “button-push” auction or simplified as a second-price auction. The button push-auction requires each buyer to keep a thumb on a button to signify their continued participation in the auction as the price, shown on an electronic board, escalates. A buyer that releases the button at any time or who fails to have their button depressed at the opening is precluded from bidding later. Within the sequential setting, the button-push auction is strategically equivalent to the second-price auction when buyers’ valuations are independently drawn.<sup>17</sup> Modeling the English auction in such a way as to allow bidders who were unwilling to bid at the opening to enter the bidding later, breaks the strategic equivalence between the English auction and the second-price auction.

Studying this more realistic model of the English auction, we see that in the sequential setting, the English auction induces bids from a larger set of buyer types than does the second-price auction. The difference is in the participation of *interim bidders*. In response, the seller in the sequential English auction model lowers the sequence of starting prices from what it would have been in the second-price auction game. The sequence of starting prices is set in such a way as to equalize revenues across the two formats as well as to make the probability of sale and the allocation of the item identical. Since the participation of interim bidders in the English auction allows the seller to lower her starting price while keeping revenues unchanged, one could say that the interim bidders are “doing the seller’s bidding.”

This paper also contributes to the literature on the Coase conjecture by extending

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<sup>17</sup>Izmalkov (2004) shows that the button-push auction is not equivalent to an English auction in which bidders may enter after the start of the auction, or re-enter, when buyers are asymmetrically informed and valuations are not independent.

the result from the monopoly setting and the bargaining setting to the auction setting. In the second-price and first-price auctions considered by M&V, the seller's initial reserve price is (generally) bounded away from the minimum valuation type.<sup>18</sup> In the English ascending-price auction game considered herein, as the time to relisting becomes short, the initial starting price converges to the minimum valuation type.

A parametric example further highlights the differences between the English and second-price auction games when the time to relisting is quite short. In particular, when the discount factor is close to unity, the seller in the English auction begins with a low starting price and subsequently drops it in small increments. In contrast, the seller in the second-price auction begins with a higher reserve, which is subsequently reduced in larger jumps. It is under these conditions that the strong start-price end-price gap is shown to result in the English auction game whereas it does not in the second-price auction game.

From an empirical standpoint, we should be interested in testing whether fishing for the opening bid is responsible for the price dynamic described by Cassady and not some form of "irrational exuberance." The model provides a testable prediction relating the size of the increment used by the seller to lower the opening bid and the comparative probability of having a subsequent auction end in a price of at least  $S$  after the initial auction started at  $S$  fails to sell. From our comparison of the English and second-price auctions, we see that the smaller is the increment, the more likely is a subsequent auction to achieve a price of  $S$ . The empirical investigation of this relationship is left to future research.

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<sup>18</sup>The exception to this is in cases where the initial screening level is equal to the minimum valuation type. This was seen in the parametric example where  $a \geq 1$ .

# APPENDIX

## A Proofs

### A.1 Proof of Lemma 1

The proof of parts 1 and 2 closely follow that of Lemma 0 of McAfee and Vincent (1997).

1. Fix a starting price  $s_t$  and for bidder 1, say, let  $dB_1$  denote the density of the highest of maximum bid prices of all other  $n - 1$  buyers should buyer 1 submit a bid. Upon bidding at the starting price, the auction will necessarily result in a sale. The expected return to bidder 1 of playing a strategy of bidding up to some amount  $b$  is

$$(v - s_t) \int_0^{s_t} dB_1 + \int_{s_t}^b (v - B_1) dB_1,$$

where we let  $v$  denote buyer 1's valuation. This expression is maximized at  $b = v$  for any set of strategies giving rise to the arbitrary density  $dB_1$ .

2. The proof proceeds to show that if some type  $v > s_t$  is an initial bidder, then so too is any type  $v' > v$ . Assuming that buyer 1, upon bidding at the starting price, bids up to  $v$  and let  $dB_1$  denote the highest of maximum bid prices of all other  $n - 1$  bidders in the current period. Let  $V_B(z; v, H_t)$  denote the continuation payoff of a type  $v$  buyer from the following period on, given history  $H_t$ , playing the strategy of a type- $z$  buyer in what follows. Further, let  $dA_1$  denote the density of the highest of maximum bid prices of all other buyers should buyer 1 abstain from bidding. If a type- $v$  buyer is an initial buyer in period  $t$ , then

$$(v - s_t) \int_0^{s_t} dB_1 + \int_{s_t}^v (v - B_1) dB_1 \geq \delta V_B(v; v, H_t) \int_0^{s_t} dA_1. \quad (8)$$

Equation (8) states that the surplus from bidding up to the buyer's valuation must exceed the expected value of not bidding.

Now suppose, by way of contradiction, that some type  $v' > v$  finds it unprofitable to bid at the start price in period  $t$ . This implies that

$$(v' - s_t) \int_0^{s_t} dB_1 + \int_{s_t}^v (v' - B_1) dB_1 < \delta V_B(v'; v', H_t) \int_0^{s_t} dA_1. \quad (9)$$

Since a type  $v$  buyer can always adopt the strategy of type  $v'$ , it must be the case that

$$\begin{aligned} V_B(v; v, H_t) &\geq V_B(v'; v, H_t) \\ &= \sum_{j=0}^{\infty} \delta^j \alpha_{t+1+j}(v') [v - m_{t+1+j}(v')], \end{aligned}$$

where  $\alpha_{t+1+j}(v'; H_t)$  denotes the probability, conditional on  $H_{t+j}$ , that the item is obtained in period  $t + 1 + j$  playing the strategy of type  $v'$  and  $m_{t+1+j}(v'; H_t)$  is the analogous expected payment. It follows that

$$V_B(v'; v', H_t) - V_B(v; v, H_t) \leq (v' - v) \sum_{j=0}^{\infty} \delta^j \alpha_{t+1+j}(v'; H_t). \quad (10)$$

From (8) and (9), we have that

$$\begin{aligned} (v' - v) \int_0^v dB_1 &< \delta [V_B(v'; v', H_t) - V_B(v; v, H_t)] \int_0^{s_t} dA_1 \\ &< (v' - v) \delta \sum_{j=0}^{\infty} \delta^j \alpha_{t+1+j}(v'; H_t) \int_0^{s_t} dA_1, \end{aligned} \quad (11)$$

where the second inequality follows from (10). Equation (11) necessarily leads to a contradiction as long as  $\int_0^v dB_1 \geq \int_0^{s_t} dA_1$  since  $\sum_{j=0}^{\infty} \delta^j \alpha_{t+1+j}(v'; H_t)$  can be no greater than 1.

Now  $\int_0^v dB_1$  is the probability of obtaining the item for the type- $v$  buyer and  $\int_0^{s_t} dA_1$



is the probability that the item goes unsold when the buyer in question abstains from bidding. So too,  $\int_0^{s_t} dB_1$  is the probability of obtaining the item for a type- $s_t$  buyer. Since a type  $s_t$  buyer wins only when he is the lone bidder, we have that

$$\int_0^{s_t} dB_1 = \int_0^{s_t} dA_1. \quad (12)$$

It follows from (12), that if we choose  $v$  to be some increment greater than  $s_t$  and increase the upper integrand on the left-hand side of (12) by that increment, we have that  $\int_0^v dB_1 \geq \int_0^{s_t} dA_1$ .

Since  $v'$  was chosen arbitrarily, it must be the case that if some type  $v$  submits a bid in period  $t$ , then so does every buyer whose valuation exceeds  $v$ .

3. We begin by asserting that there exists a minimum starting price such that all bidder types bid whenever the starting price is less than or equal to the minimum, regardless of the history. I claim that  $\underline{v}-\bar{v}$  is one such starting price. We know that in equilibrium, the seller's expected receipts must be nonnegative—since she can always opt not to sell—and that a buyer's expected surplus cannot exceed  $\bar{v}$  by the same token. Therefore, the expected surplus for a buyer with valuation  $\underline{v}$  is *at most*  $\underline{v}$  minus the starting price. This is less than  $\bar{v}$  as long as the starting price is less than  $\underline{v}-\bar{v}$ . Thus, all types bid when the starting price is less than or equal to  $\underline{v}-\bar{v}$ .

We now calculate a buyer's expected surplus at the minimum starting price. When all buyer types bid and the starting price is less than  $\underline{v}$ , a given buyer's expected surplus is  $\int_{\underline{v}}^v F_{Y_1}(y) dy \geq 0$ . Notice that a buyer's expected surplus is independent of the actual starting price as the price will necessarily be determined by the bid of the second-highest valuation buyer. This is crucial in what follows.

We now use recursive logic to show that the minimum starting price is in fact  $\underline{v}$ . Consider a starting price,  $s_\varepsilon = \underline{v}-\bar{v} + \varepsilon$ , just slightly greater than  $\underline{v}-\bar{v}$ , such that if the auction started at  $s_\varepsilon$  fails to sell, the starting price is reduced to  $\underline{v}-\bar{v}$  in the following

period. When the starting price is  $s_\varepsilon$ , a given buyer bids at the start as long as the surplus gained in the current period exceeds the surplus gained in the following period should the item go unsold. Assume by way of contradiction that there exists some  $\beta_\varepsilon > \underline{v}$  that is the lowest type to bid at the starting price. The payoff to bidding for some valuation- $v$  buyer, in the current period, is

$$\Pi(\beta_\varepsilon) = [\beta_\varepsilon - \rho(s_\varepsilon, \beta_\varepsilon)] F_{Y_1}(\beta_\varepsilon) + \int_{\beta_\varepsilon}^v F_{Y_1}(Y_1) dY_1,$$

where  $\rho(s_\varepsilon, \beta_\varepsilon)$  denotes expected payment conditional on being the lone initial bidder, taking into account the bidding of interim bidders. Since the value of waiting for the following period is  $\delta \int_{\underline{v}}^v F_{Y_1}(Y_1) dY_1$ , a buyer with valuation  $v > \beta_\varepsilon$  bids in the following period as long as

$$[\beta_\varepsilon - \rho(s_\varepsilon, \beta_\varepsilon)] F_{Y_1}(\beta_\varepsilon) + \int_{\beta_\varepsilon}^v F_{Y_1}(Y_1) dY_1 \geq \delta \int_{\underline{v}}^v F_{Y_1}(Y_1) dY_1. \quad (13)$$

From Lemma 1, if this condition holds for the lowest buyer type  $\beta_\varepsilon$ , it holds for all higher types. Thus it is sufficient to show that this condition holds for  $v = \beta_\varepsilon$ , for which it is sufficient that,

$$\int_{\underline{v}}^{\beta_\varepsilon} F_{Y_1}(Y_1) dY_1 \geq \delta \int_{\underline{v}}^{\beta_\varepsilon} F_{Y_1}(Y_1) dY_1. \quad (14)$$

Equation (14) follows from (13) where the left-hand side of (13) has been minimized by setting  $v$  equal to its minimum  $\beta_\varepsilon$  and  $\rho(s_\varepsilon, \beta_\varepsilon)$  equal to  $\int_{\underline{v}}^{\beta_\varepsilon} Y_1 dF_{Y_1} / F_{Y_1}(\beta_\varepsilon)$  its maximum. This condition clearly holds for all  $\beta_\varepsilon \geq \underline{v}$  which contradicts the assumption that  $\beta_\varepsilon > \underline{v}$ , so we conclude that all buyer types bid when the starting price is  $s_\varepsilon$  or less.

Now consider a starting price  $s_{\varepsilon_1} = -\bar{v} + \varepsilon_1$ ,  $\varepsilon_1 > \varepsilon$  such that if the item fails to sell at  $s_{\varepsilon_1}$  the seller reduces the starting price to  $s_\varepsilon$  in the following period. When the seller sets a starting price of  $s_{\varepsilon_1}$  in the current period, buyers bid at the start only if the payoff

from doing so exceeds the payoff of waiting for the following period. Assume by way of contradiction that there exists some  $\beta_{\varepsilon_1} > \underline{v}$  that is the lowest type to bid. Since all buyer types bid when the starting price is  $s_{\varepsilon}$ , a buyer with valuation  $v > \beta_{\varepsilon_1}$  bids when the starting price is  $s_{\varepsilon_1}$  as long as

$$[\beta_{\varepsilon_1} - \rho(s_{\varepsilon_1}, \beta_{\varepsilon_1})] F_{Y_1}(\beta_{\varepsilon_1}) + \int_{\beta_{\varepsilon_1}}^v F_{Y_1}(Y_1) dY_1 \geq \delta \int_{\underline{v}}^v F_{Y_1}(Y_1) dY_1.$$

As before, the left-hand side can be minimized with  $v$  equal to  $\beta_{\varepsilon_1}$  and  $\rho(s_{\varepsilon_1}, \beta_{\varepsilon_1})$  equal to  $\int_{\underline{v}}^{\beta_{\varepsilon_1}} Y_1 dF_{Y_1} / F_{Y_1}(\beta_{\varepsilon_1})$ , leading to condition (14) only with  $\beta_{\varepsilon_1}$  playing the role of  $\beta_{\varepsilon}$ . This condition holds for all  $\beta_{\varepsilon_1} \geq \underline{v}$ , once again contradicting the assumption that  $\beta_{\varepsilon_1} > \underline{v}$ . We then conclude that all types bid when the starting price is  $s_{\varepsilon_1}$  or less.

Continuing recursively in this manner, we see that for any  $s_{\varepsilon_k} \leq \underline{v}$  such that all buyer types bid whenever the starting price is  $s_{\varepsilon_k}$  or less, for an arbitrary  $k$ , then all types also bid when the starting price is  $s_{\varepsilon_{k+1}} \in [s_{\varepsilon_k}, \underline{v}]$ . This establishes the result. Note that the recursion does not extend to  $s_{\varepsilon_{k+1}} > \underline{v}$  since such starting prices may actually determine the price with the consequence that a buyer's participation decision does not give rise to equation (14).

4. Let  $g(u_t, \beta_t, \beta_{t+1})$  denote the seller's expected return with beliefs  $u_t$ , when choosing a starting price in the current period that induces a screening level of  $\beta_t$ , which subsequently induces a screening level of  $\beta_{t+1}$  in the following period. Note that from part 2 of the lemma,  $\beta_t$  exceeds  $\beta_{t+1}$  and from part 3,  $\beta_{\tau}$  is equal to  $\underline{v}$  if  $s_{\tau} \leq \underline{v}$ . We have that

$$\begin{aligned} g(u_t, \beta_t, \beta_{t+1}) &= n\rho(s_t, \beta_t) [F(u_t) - F(\beta_t)] F_{Y_1}(\beta_t) \\ &\quad + n \int_{\beta_t}^{u_t} \int_{\beta_t}^{X_1} Y_1 dF_{Y_1} f(X_1) dX_1 \\ &\quad + \delta \Gamma_{t+1}(\beta_t), \end{aligned}$$

where  $\Gamma_{t+1}(\beta_t)$  is the seller's optimal payoff from period  $t + 1$  on, beginning at state  $\beta_t$ . The first-order condition for the seller's optimal choice of  $\beta_t$  reduces to

$$(1 - \delta) [F(u_t) - F(\beta_t) - \beta_t f(\beta_t)] F_{Y_1}(\beta_t) \leq 0.$$

Since  $f(\cdot)$  is positive, there exists some  $u^* > \underline{v}$  such that for  $u_t < u^*$ ,  $F(u_t) - F(\beta_t) - \beta_t f(\beta_t)$  is strictly negative. Thus for  $u < u^*$ , it is optimal for the seller to induce bids from all types thus ending the game.

Next we show that the seller's beliefs fall below  $u^*$  in finite time. For this, we again examine the seller's first-order condition for the optimal screening level. Solving for an interior optimum and rearranging terms yields

$$F(u_t) - F(\beta_t) = \beta_t f(\beta_t).$$

Since  $f$  is bounded away from zero, so too is the distance between  $u_t$  and  $\beta_t$ . So in an interior optimum, implying  $u_t > u^*$ , the screening level jumps down in discrete steps so that some  $u^* > \underline{v}$  is eventually reached. If the optimum is not interior, then by definition,  $u_t < u^*$ .

## A.2 Proof of Proposition 1

We begin by defining a  $k$ -period game in which the seller and each buyer behaves optimally given the constraint that should the item fail to sell, the game necessarily ends after  $k - 1$  periods. For this constrained game, denote the screening level, seller's starting price, and expected revenue in the terminal period:

$$\beta_0 \equiv \underline{v}, s_0 \equiv \underline{v}, \Gamma_0(u) \equiv n \int_{\underline{v}}^u \int_{\underline{v}}^{X_1} Y_1 dF_{Y_1} f(X_1) dX_1.$$

Expected revenue is calculated by considering first the expected payment of some buyer, say buyer 1, with valuation  $X_1$ . Since the auction in the final period is run with a starting price of  $\underline{v}$ , buyer 1's price will be determined by the maximum of  $n - 1$  valuations of other buyers, denoted  $Y_1$ . Buyer 1's expected payment is the expectation of  $Y_1$  over  $[\underline{v}, X_1]$ , where  $F_{Y_1} \equiv F^{n-1}$  denotes the distribution of  $Y_1$ . The seller's expected revenue is simply  $n$ -times the expectation of a given buyer's expected payment.

Define the sequences

$$\{\beta_j\}_{j=0}^k, \{\sigma_j\}_{j=0}^k, \{\Gamma_j\}_{j=0}^k, \{g_j\}_{j=0}^k$$

iteratively in what follows.

Let  $\sigma_j(x)$  denote the starting price that induces a screening level of  $x$  in the  $j$ th-to-last period of the  $k$ -period game. Since the type- $x$  buyer wins the auction only upon being the lone bidder,  $\sigma_j(x)$  satisfies

$$[x - \rho(\sigma_j(x), x)] F_{Y_1}(x) = \delta \left( [\beta_{j-1} - \rho_{j-1}] F_{Y_1}(\beta_{j-1}) + \int_{\beta_{j-1}}^x F_{Y_1}(Y_1) dY_1 \right) \quad (15)$$

The left-hand side of the expression indicates a type- $x$  buyer's expected surplus from bidding at starting price  $\sigma_j(x)$ , when he is the lowest type to be an initial bidder. His expected payment under the circumstance is  $\rho(\sigma_j(x), x)$ . In the SPA game, absent interim bidders, his payment would be  $\sigma_j$ . However, since a bid at the starting price induces bids from those who otherwise would have waited for a subsequent period prior to bidding, his payment goes up. The term  $\rho(\sigma_j(x), x)$  represents the expected maximum bid price of all interim bidders, given that such a bid is no greater than  $x$ . Since all interim bidders are indifferent between not bidding and bidding at any price up to their valuations, the determination of  $\rho$  is completely arbitrary. We assume in what follows that the function  $\rho(\cdot, \cdot)$ , strictly increasing in both arguments, is known to all players

and that each buyer, when contemplating the decision of whether to bid at the reserve, uses the same calculation of  $\rho$ .

The right-hand side of the expression gives a type- $x$  buyer's expected continuation surplus, given a starting price of  $\sigma_{j-1}$  and a screening level of  $\beta_{j-1}$  in the period to follow. In the following period, since  $\beta_j \geq \beta_{j-1}$ , he receives an amount given by the first term, where  $\rho_j \equiv \rho(\sigma_{j-1}, \beta_{j-1})$ , in the event that he is the lone initial buyer and an amount given by the second term when he is bidding against at least one other initial bidder.

Let  $g_j(u, x)$  denote the seller's revenue in the  $j$ th-to-last period, at state  $u$ , when choosing a starting price that induces a screening level of  $x$ . We have that

$$\begin{aligned} g_j(u, x) = & n\rho(\sigma_j(x), x) F_{Y_1}(x) [F(u) - F(x)] + n \int_x^u \int_x^{X_1} Y_1 dF_{Y_1} f(X_1) dX_1 \\ & + \delta \Gamma_{j-1}(x). \end{aligned} \quad (16)$$

The first term represents the seller's revenue from having only one initial buyer and the second from having at least two. The third term represents her maximum discounted return from the following period on, at state  $x$ , in the event the current auction fails to produce a sale. Therefore,

$$\Gamma_j(u) = \max_{x \leq u} g_j(u, x) \quad (17)$$

and

$$\beta_j = \arg \max_{x \leq u} \{g_j(u, x)\}. \quad (18)$$

**Lemma 4** *For a given  $k > 1$ , the sequences  $\{\beta_j\}_{j=0}^k$ ,  $\{\rho_j\}_{j=0}^k$ ,  $\{\Gamma_j\}_{j=0}^k$  are such that:*

1. *The  $\sigma_j < x$  satisfying (15) are unique and increasing in  $x$  for  $x > \underline{v}$ .*
2. *The  $\Gamma_j(x)$  satisfying (17) are increasing and continuous.*

3. The  $\beta_j(u)$  satisfying (18) are strictly less than  $u$  and increasing.

**Proof.** Property 1 is proven directly from (15). Uniqueness follows from the fact that the left-hand side of (15) is strictly decreasing in  $\sigma$  while the right-hand side is constant in  $\sigma$  for  $x > \underline{v}$ . Differentiating both sides of (15) with respect to  $x$  yields

$$\frac{\partial \rho(\sigma_j, x)}{\partial \sigma_j} \frac{d\sigma_j}{dx} F_{Y_1}(x) = (1 - \delta) F_{Y_1}(x) + [x - \rho_j(\sigma_j, x)] f_{Y_1}(x).$$

The right-hand side of this expression is positive by the fact that  $\rho_j \leq x$ .

We prove properties 2 and 3 by induction. It is straightforward to show that properties 2 and 3 are satisfied for  $j = 2$ . Now assume, by way of induction, that 2 and 3 are satisfied for  $j = k - 1$ . Since  $\rho(\sigma_k(x), x)$  is continuous in both arguments and  $\sigma_k(x)$  is continuous in  $x$ , then  $g_k(u, x)$  is continuous in both arguments. It follows from standard arguments that  $\Gamma_k$  is continuous and increasing.

For property 3, consider  $u < u'$  and let  $x \in \arg \max_y \{g_k(u, y)\}$  and let  $x' \in \arg \max_y \{g_k(u', y)\}$ . Suppose, by way of contradiction, that  $x' < x$ . We have that

$$\begin{aligned} g_k(u', x) &= g_k(u, x) + n\rho(\sigma_k(x), x) F_{Y_1}(x) [F(u') - F(u)] \\ &\quad + n \int_u^{u'} \int_x^{X_1} Y_1 dF_{Y_1}(Y_1) f(X_1) dX_1 \end{aligned} \quad (19)$$

and

$$\begin{aligned} g_k(u', x') &= g_k(u, x') + n\rho(\sigma_k(x'), x') F_{Y_1}(x') [F(u') - F(u)] \\ &\quad + n \int_u^{u'} \int_{x'}^{X_1} Y_1 dF_{Y_1}(Y_1) f(X_1) dX_1. \end{aligned} \quad (20)$$

Subtracting (20) from (19), we have

$$g_k(u', x) - g_k(u', x') - [g_k(u, x) - g_k(u, x')] = n \left[ \rho(\sigma_k(x), x) F_{Y_1}(x) - \rho(\sigma_k(x'), x') F_{Y_1}(x') - \int_{x'}^x Y_1 dF_{Y_1}(Y_1) \right] [F(u') - F(u)]. \quad (21)$$

The left-hand side of (21) is non-positive since  $x$  is a maximizer of  $g_k(u, \cdot)$  and  $x'$  is a maximizer of  $g_k(u', \cdot)$ . We now want to show the right-hand side of (21) to be positive, so resulting in a contradiction.

For ease of notation, let  $\Upsilon \equiv \rho(\sigma_k(x), x) F_{Y_1}(x)$  and  $\Upsilon' \equiv \rho(\sigma_k(x'), x') F_{Y_1}(x')$ .

Using this notation, the right-hand side of (21) is positive if

$$\Upsilon - \Upsilon' - \int_{x'}^x Y_1 dF_{Y_1}(Y_1) \geq 0. \quad (22)$$

From the period- $k$  analogue of (16), we have

$$\Upsilon = (1 - \delta) x F_{Y_1}(x) + \delta \Upsilon_{k-1} + \delta \int_{\beta_{k-1}}^x Y_1 dF_{Y_1}(Y_1),$$

where  $\Upsilon_{k-1} \equiv \rho(\sigma_{k-1}(\beta_{k-1}), \beta_{k-1}) F_{Y_1}(\beta_{k-1})$  and  $\beta_{k-1} \equiv \beta_{k-1}(x)$ . Using the fact that, from (16),  $d\Upsilon/d\beta_{k-1} \geq 0$ ,

$$\begin{aligned} \Upsilon &\geq (1 - \delta) x F_{Y_1}(x) + \delta \Upsilon'_{k-1} + \delta \int_{\beta'_{k-1}}^x Y_1 dF_{Y_1}(Y_1) \\ &= \Upsilon' + (1 - \delta) [x F_{Y_1}(x) - x' F_{Y_1}(x')] + \delta \int_{x'}^x Y_1 dF_{Y_1}, \end{aligned}$$

where  $\beta'_{k-1} < \beta_{k-1}$  and so too  $\Upsilon'_{k-1} < \Upsilon_{k-1}$  by the assumption that  $x' < x$ . It follows



that (22) holds if

$$(1 - \delta) \left( [xF_{Y_1}(x) - x'F_{Y_1}(x')] - \int_{x'}^x Y_1 dF_{Y_1} \right) \geq 0.$$

The above is true under the assumption that  $x' < x$ , thus yielding the desired contradiction. ■

Having characterized the equilibrium to the arbitrarily constrained  $k$ -period game, we can extend the results of Lemma 4 to the unconstrained game. In fact, we show that at any stage of the game, the number of remaining periods is determined solely by  $u$ , the highest potential buyer valuation. This is done by constructing a sequence of numbers  $\{z_j\}_{j=0}^T$  iteratively as follows. Let

$$z_1 = \sup \{u | \beta_1(u) = \underline{v}\}$$

denote the largest value of  $u$  such that the seller chooses to end the game immediately and

$$z_j = \min \left\{ \sup \{u | \beta_j(u) \leq z_{j-1}\}, \bar{v} \right\}$$

denote the largest value of  $u$  such that the seller chooses a screening level in the current period such that the optimal policy from the following period onward has her end the game in  $j - 1$  periods. The proof of Part 4 of Lemma 1 demonstrated that  $z_1$  is the largest value of  $u$  such that  $F(u_t) - F(\beta_t) - \beta_t f(\beta_t)$  is negative. The following Lemma formalizes the argument and shows that there exists some  $T$  such that when  $u = \bar{v}$ , the seller chooses a screening level such that the optimal policy from the following period onward has her end the game in  $T - 1$  periods.

**Lemma 5** *There exists an  $\varepsilon > 0$  such that for all  $\rho$ ,  $\delta$ , and  $n$ ,  $z_1 \geq 1 + \varepsilon$ . Further, there exists a  $T < \infty$  such that  $z_T = \bar{v}$ .*

The proof is identical to that of Lemma 2 in M&V, only with  $\rho$  taking the place of  $\sigma$ , so there is no need to repeat it here.

With the  $z_j$  so defined, we can define the seller's problem uniquely by  $u$ , independent of  $j$ . In this way, if  $u \in (z_{j-1}, z_j]$ , the seller chooses the optimal screening level independent of  $j$ ; it just so happens that such a screening level will lead, assuming optimal behavior in what follows, to the game ending in  $j - 1$  more periods should the item fail to sell. In what follows, we change our notational convention so that a subscript  $t$  denotes  $(t - 1)$  periods *after* the initial period as opposed to  $t$  periods *before* the terminal period. In this way, given  $u_1 = \bar{v}$ , we have that  $u_2 = \beta(\bar{v})$ ,  $u_t = \beta(u_{t-1})$  and  $s_t = \sigma(\beta_t)$  for any  $t > 1$ .

The following addresses the three individual components of Proposition 1.

1. Using the same techniques developed in the solution to the  $k$ -period constrained problem, we solve for the PBE sequence of screening levels and corresponding starting prices through backward induction. By construction, the screening level in the terminal period is  $\underline{v}$ . In any period prior to the last, the seller chooses  $\beta_t$  to maximize  $g(u, x)$  subject to the sequential-rationality constraint imposed by equation (1). This gives rise to the first-order condition

$$\beta_t f(\beta_t) + F(\beta_t) - F(u_t) \leq 0$$

which characterizes a solution in  $u$ , independent of  $n$ ,  $\delta$ , and  $\rho$ .

2. For a given sequence of screening levels,  $\{\rho_t\}$  is the sequence of the expected payment made by the marginal bidder type in each period. Using (1),  $\rho_{T-1}$  satisfies

$$(\beta_{T-1} - \rho_{T-1}) F_{Y_1}(\beta_{T-1}) = \delta \int_{\underline{v}}^{\beta_{T-1}} F_{Y_1} dY_1.$$

In any period  $t < T - 1$ ,  $\rho_t$  satisfies

$$(\beta_t - \rho_t) F_{Y_1}(\beta_t) = \delta \left( [\beta_{t+1} - \rho_{t+1}] F_{Y_1}(\beta_{t+1}) + \int_{\beta_{t+1}}^{\beta_t} F_{Y_1}(Y_1) dY_1 \right), \quad (23)$$

so that the sequence  $\{\rho_t\}$  is unique to a given sequence of  $\{\beta_t\}$ . The payment by the marginal type is a known function,  $\rho(\cdot, \cdot)$  of the starting price and screening level. Therefore, for the optimal sequence of screening levels  $\{\beta_t\}$  and the corresponding sequence of payments  $\{\rho_t\}$ , an equilibrium starting price  $\sigma(\beta_t)$  is some price such that  $\rho(\sigma(\beta_t), \beta_t) = \rho_t$ . Under the assumption that  $\rho$  be increasing in both arguments,  $\sigma(\beta_t)$  is unique and increasing in  $\beta_t$ .

3. Since the seller continues to relist until a sale is transacted, for any value of  $\rho$ , the item is allocated to the buyer with the highest valuation. The probability of sale in some period  $t$  is  $F(\beta_{t-1}) - F(\beta_t)$ . Since the sequence of screening levels is the same for each value of  $\rho$ , so is the probability of sale in each period. The seller's revenue in a given period is given by the first two terms in  $g(u_t, \beta_t)$ . Using the fact that  $u_t = \beta_{t-1}$ , after some simplification, we have the seller's period- $t$  revenue is

$$\begin{aligned} R(\beta_{t-1}, \beta_t) &= n \int_{\beta_t}^{\beta_{t-1}} [vf(v) + F(v) - F(\beta_{t-1})] F_{Y_1}(v) dv \\ &\quad - n [F(\beta_{t-1}) - F(\beta_t)] (\beta_t - \rho_t) F_{Y_1}(\beta_t). \end{aligned}$$

Substituting in from (23) recursively  $(T - t)$  times, we have

$$\begin{aligned} R(\beta_{t-1}, \beta_t) &= n \int_{\beta_t}^{\beta_{t-1}} [vf(v) + F(v) - F(\beta_{t-1})] F_{Y_1}(v) dv \\ &\quad - n [F(\beta_{t-1}) - F(\beta_t)] \sum_{j=0}^{T-t} \delta^{j+1} \int_{\beta_{t+j+1}}^{\beta_{t+j}} F_{Y_1}(Y_1) dY_1, \end{aligned}$$

which is independent of  $\rho$ .

### A.3 Proof of Lemma 2

The effect of the trembles on an interim bidders is to create the possibility of winning the auction when bidding no more than his valuation. This is because 1. the buyer who bid at the starting price may simply be an interim bidder who bid by mistake; or 2. all true initial bidders may mistakenly drop out of the bidding before their valuations are reached. It is sufficient to show that given the possibility of obtaining the item, all interim bidders' strategies other than the one proposed are weakly dominated.

Consider bidder 1, an interim bidder with valuation  $v \geq s_t$ , and let  $dB_\varepsilon$  denote the density of the highest maximum bid price of all  $n - 1$  other buyers, given that a bid was placed at the starting price. Note that if the probability of all trembles were zero to be zero,  $dB_\varepsilon$  would be equal to  $dF_{Y_1}$ . Suppose buyer 1 chooses some maximum price  $b$  at which to drop out. His expected payoff from that strategy is

$$\int_{s_t}^b (v - B_\varepsilon) dB_\varepsilon.$$

This expression is maximized at  $b = v$  for any profile of behavioral strategies played by the other bidders which gives rise to  $dB_\varepsilon$ .

### A.4 Proof of Proposition 2

Suppose the seller wishes to induce a screening level of  $\beta_1$  in the initial period. If  $\beta_1 = \underline{v}$ , then she simply chooses a starting price no greater than  $\underline{v}$  and we're done. Assume then that  $\beta_1 > \underline{v}$ . To induce  $\beta_1$ , the seller chooses a reserve  $\sigma(\beta_1)$  giving rise to  $\rho_1 \equiv \rho(\sigma(\beta_1), \beta_1)$ , solving

$$(\beta_1 - \rho_1) F_{Y_1}(\beta_1) = \delta \left[ \int_{\beta_2}^{\beta_1} F_{Y_1} dY_1 + (\beta_2 - \rho_2) F_{Y_1}(\beta_2) \right], \quad (24)$$

where  $\beta_2 = \beta(\beta_1)$  and  $\rho_2 = \rho(\sigma(\beta_2), \beta_2)$ . By the same logic, the second term on the right-hand side of (24), assuming  $\beta_2 > \underline{v}$  satisfies

$$(\beta_2 - \rho_2) F_{Y_1}(\beta_2) = \delta \left[ \int_{\beta_3}^{\beta_2} F_{Y_1} dY_1 + (\beta_3 - \rho_3) F_{Y_1}(\beta_3) \right].$$

Following this logic recursively, and noting that

$$(\beta_{T-1} - \rho_{T-1}) F_{Y_1}(\beta_{T-1}) = \delta \int_{\underline{v}}^{\beta_{T-1}} F_{Y_1} dY_1,$$

since  $\beta_T = \underline{v}$ , (24) becomes

$$(\beta_1 - \rho_1) F_{Y_1}(\beta_t) = \delta \sum_{j=0}^{T-(t+1)} \delta^j \int_{\beta_{t+j+1}}^{\beta_{t+j}} F_{Y_1} dY_1. \quad (25)$$

Using (2) on the left-hand side of (25),

$$(\beta_1 - \rho_1) F_{Y_1}(\beta_t) = \int_{\sigma(\beta_1)}^{\beta_1} F_{Y_1} dY_1. \quad (26)$$

We are interested in the value of  $\sigma(\beta_1)$  as  $\delta$  gets arbitrarily close to unity. Therefore, in (25),

$$\lim_{\delta \rightarrow 1} \delta \sum_{j=0}^{T-(t+1)} \delta^j \int_{\beta_{t+j+1}}^{\beta_{t+j}} F_{Y_1} dY_1 = \int_{\underline{v}}^{\beta_1} F_{Y_1} dY_1. \quad (27)$$

Putting (26) together with (27), (25) implies

$$\lim_{\delta \rightarrow 1} \int_{\sigma(\beta_1)}^{\beta_1} F_{Y_1} dY_1 = \int_{\underline{v}}^{\beta_1} F_{Y_1} dY_1.$$

The only way this can hold is if  $\sigma(\beta_1) \rightarrow \underline{v}$ .

## A.5 Proof of Proposition 3

In the commitment equilibrium, the seller chooses a sequence of screening levels  $\{\hat{\beta}_t\}_{t=1}^{\hat{T}}$  to maximize the discounted revenue stream subject to sequential rationality constraints. Of course, in order to be talking about screening levels, one must first prove the successive skimming property for the commitment solution. The proof is identical to that of the no-commitment equilibrium and so is omitted here. We begin by considering a two period problem and characterize the solution. It is then straightforward to extend the results of the two-period game to a game of arbitrary duration.

Fix the sequence of screening levels  $\{\hat{\beta}_1, \hat{\beta}_2\}$  and consider the starting prices required to induce such an outcome under sequential rationality. As in the no-commitment equilibrium, it is straightforward to show that conditional on bidding, an initial bidder bids his valuation. In period 2, a buyer is an initial bidder as long as  $s_2 \leq v$ . Therefore  $\hat{\beta}_2 = s_2 = \rho_2$ . Assuming a period-2 starting price of  $s_2$ , the value of  $s_1$  inducing a screening level of  $\beta_1 \geq s_2$  satisfies

$$(\hat{\beta}_1 - \rho_1) F_{Y_1}(\hat{\beta}_1) = \delta \int_{s_2}^{\hat{\beta}_1} F_{Y_1}(Y_1) dY_1,$$

where  $\rho_1 \equiv \rho(s_1, \hat{\beta}_1)$  as in the no-commitment equilibrium.

Given the sequence  $\{\hat{\beta}_1, \hat{\beta}_2\}$ , the seller's period-1 receipts are

$$\begin{aligned} R(\bar{v}, \hat{\beta}_1) &= n \left[ 1 - F(\hat{\beta}_1) \right] F_{Y_1}(\hat{\beta}_1) + n \int_{\hat{\beta}_1}^{\bar{v}} \int_{\hat{\beta}_1}^{X_1} Y_1 dF_{Y_1} f(X_1) dX_1 \\ &= n \int_{\hat{\beta}_1}^{\bar{v}} [v f(v) + F(v) - 1] F_{Y_1}(v) dv \\ &\quad - n \left[ 1 - F(\hat{\beta}_1) \right] (\hat{\beta}_1 - \rho_1) F_{Y_1}(\hat{\beta}_1). \end{aligned}$$

where the term

$$n \int_{\hat{\beta}_1}^{\bar{v}} [v f(v) + F(v) - 1] F_{Y_1}(v) dv$$

is a seller's revenue in a one-shot auction where the highest type is  $\bar{v}$  and the starting price is  $\hat{\beta}_1$ . This expression is maximized at  $s^*$  such that

$$s^* f(s^*) + F(s^*) - 1 = 0.$$

As this expression plays an important role in what follows, we define  $\phi(v)$  such that

$$\phi(v) = n[vf(v) + F(v) - 1]F_{Y_1}(v).$$

Since the game is artificially constrained to end after period 2, the period-2 auction is like a one-shot auction where the highest type is  $\hat{\beta}_1$ . As such, the (unconditional) expected revenues are  $\int_{s_2}^{\hat{\beta}_1} \phi(v) dv$ . The seller then chooses  $\hat{\beta}_1$  and  $s_2$  to solve

$$\max_{\hat{\beta}_1, s_2} \int_{\hat{\beta}_1}^{\bar{v}} \phi(v) dv - n[1 - F(\hat{\beta}_1)](\hat{\beta}_1 - \rho_1)F_{Y_1}(\hat{\beta}_1) + \int_{s_2}^{\hat{\beta}_1} \phi(v) dv.$$

$$\text{subject to } (\hat{\beta}_1 - \rho_1)F_{Y_1}(\hat{\beta}_1) = \delta \int_{s_2}^{\hat{\beta}_1} F_{Y_1}(Y_1) dY_1$$

$$\text{and } \bar{v} \geq \hat{\beta}_1 \geq s_2 \geq \underline{v}.$$

Since the sequential rationality constraint is satisfied with equality, we can substitute it into the objective function. After rearranging terms, the seller's problem becomes

$$\max_{\hat{\beta}_1, s_2} (1 - \delta) \int_{\hat{\beta}_1}^{\bar{v}} \phi(v) dv + \delta \int_{s_2}^{\bar{v}} \phi(v) dv$$

$$\text{subject to } \bar{v} \geq \hat{\beta}_1 \geq s_2 \geq \underline{v}.$$

This expression is maximized, term by term, by setting  $\hat{\beta}_1 = s_2 = s^*$ .

Now consider a solution with some arbitrary number of periods. As before, fix the sequence of screening levels  $\{\hat{\beta}_1, \hat{\beta}_2, \dots, \hat{\beta}_{\hat{T}}\}$  and consider the sequence of starting

prices required to induce the sequence of screening levels. In the final period, sequential rationality implies that a buyer bid his valuation as long as it exceeds the starting price; thus  $\hat{\beta}_{\hat{T}} = s_{\hat{T}}$ . In the second-to-last period, sequential rationality requires that  $s_{\hat{T}-1}$  satisfy

$$\left( \hat{\beta}_{\hat{T}-1} - \rho \left( s_{\hat{T}-1}, \hat{\beta}_{\hat{T}-1} \right) \right) F_{Y_1} \left( \hat{\beta}_{\hat{T}-1} \right) = \delta \int_{s_{\hat{T}}}^{\hat{\beta}_{\hat{T}-1}} F_{Y_1} (Y_1) dY_1.$$

In any period  $t < \hat{T} - 1$ , for a given sequence of starting prices  $\{s_{t+1}, \dots, s_{\hat{T}}\}$  and screening levels  $\{\hat{\beta}_{t+1}, \dots, s_{\hat{T}}\}$ , sequential rationality requires  $s_t$  satisfy

$$\left( \hat{\beta}_t - \rho \left( s_t, \hat{\beta}_t \right) \right) F_{Y_1} \left( \hat{\beta}_t \right) = \delta \left( \left( \hat{\beta}_{t+1} - \rho_{t+1} \right) F_{Y_1} \left( \hat{\beta}_{t+1} \right) + \int_{\hat{\beta}_{t+1}}^{\hat{\beta}_t} F_{Y_1} (Y_1) dY_1 \right).$$

The seller's revenue in the final period is  $\int_{s_{\hat{T}}}^{\hat{\beta}_{\hat{T}-1}} \phi(v) dv$ . In any period  $t < \hat{T}$ , her revenue is

$$R \left( u_t, \hat{\beta}_t \right) = \int_{\hat{\beta}_t}^{u_t} \phi(v) dv - n \left[ F(u_t) - F(\hat{\beta}_t) \right] \left( \hat{\beta}_t - \rho_t \right) F_{Y_1} \left( \hat{\beta}_t \right),$$

where  $u_t = \hat{\beta}_{t-1}$ . The sequence of screening levels  $\{\hat{\beta}_{t+1}, \dots, s_{\hat{T}}\}$  is chosen to solve

$$\max_{\hat{\beta}_{t+1}, \dots, s_{\hat{T}}} \sum_{t=1}^{\hat{T}} \delta^{t-1} R \left( \hat{\beta}_t, \hat{\beta}_{t+1} \right)$$

subject to sequential rationality constraints

$$\text{and } \bar{v} \geq \hat{\beta}_1 \geq \dots \geq s_{\hat{T}} \geq \underline{v}.$$

Substituting in for the sequential rationality constraints and rearranging terms in the



same manner as in the two-period game, the seller's problem becomes

$$\begin{aligned} \max_{\hat{\beta}_{t+1}, \dots, s_{\hat{T}}} (1 - \delta) \sum_{t=1}^{\hat{T}-1} \delta^{t-1} \int_{\hat{\beta}_t}^{\bar{v}} \phi(v) dv + \delta^{\hat{T}-1} \int_{s_{\hat{T}}}^{\bar{v}} \phi(v) dv \\ \text{subject to } \bar{v} \geq \hat{\beta}_1 \geq \dots \geq s_{\hat{T}} \geq \underline{v}. \end{aligned}$$

Once again, each term is individually maximized by setting  $\hat{\beta}_1 = \dots \hat{\beta}_{\hat{T}-1} = s_{\hat{T}-1} = s^*$ .

## A.6 Proof of Lemma 3

1. From (3) and (4), differentiating with respect to  $s_t$  yields:

$$\frac{\partial b_t}{\partial s_t} = F_{Y_1}(b_t) / [(1 - \delta) F_{Y_1}(b_t) + (1 - \alpha)(b_t - \rho_t) f_{Y_1}(b_t)] > 0$$

2. When  $s_t \leq \underline{v}$ ,  $\Pi_{t+1} \equiv \Pi(\beta_{t+1})$  is necessarily zero by result 3 of Lemma (1). This requires that the left-hand side of (3) also be zero, which implies  $b_t = \underline{v}$ . When  $s_t > \underline{v}$ ,  $\Pi_{t+1} > 0$ , which requires the left-hand side of (3) to be positive, implying  $b_t > \rho_t > s_t$ .

3. Differentiating (3) and (4) with respect to  $\alpha$ ,

$$\frac{\partial b_t}{\partial \alpha} = \left( \frac{\partial \rho_t}{\partial \alpha_t} \right) F_{Y_1}(b_t) / [(1 - \delta) F_{Y_1}(b_t) + (1 - \alpha)(b_t - s_t) f_{Y_1}(b_t)] > 0.$$

4. The indifference condition can be re-written as

$$b_t - \rho_t = \delta \frac{\Pi(b_t)}{F_{Y_1}(b_t)}.$$

The right-hand side can be interpreted conveniently as an expectation where

$$\frac{\Pi(\beta_t)}{F_{Y_1}(\beta_t)} = E [\beta_t - \rho_{t+1} I \{Y_1 < b_{t+1}\} - Y_1 I \{Y_1 > b_{t+1}\} | Y_1 < b_t] .$$

This is the expected period- $t + 1$  payoff for a type  $v$  buyer, conditional on the period being reached. This is smaller for larger values of  $n$  due to the stochastic dominance of a given order-statistic from a larger sample, where  $I$  represents an indicator function. Intuitively, it should follow that a buyer bidding against a larger set of competitors should be worse off. It then follows from (3), that  $b_t - \rho_t$  is smaller for larger values of  $n$ . As  $n$  goes to infinity, the probability that period  $t + 1$  is reached,  $F(b_t)^n$ , goes to zero for any value of  $b_t$ . Since the right hand side of (3) goes to zero, so too must the left. This requires that  $\beta_t = \rho_t = s_t$  in the limit.

5. From (3) and (4), differentiating with respect to  $\delta$  yields:

$$\frac{\partial b_t}{\partial \delta} = \Pi(b_t) / [(1 - \delta) F_{Y_1}(b_t) + (b_t - \rho_t) f_{Y_1}(b_t)] > 0.$$

As  $\delta$  goes to zero, so does the right-hand side of (3). This requires that  $\beta_t = \rho_t = s_t$  in the limit.

## A.7 Proof of Proposition 4

Consider period  $t + 1$  following the period- $t$  auction, with starting price  $S$  and screening level  $b(u, S; \alpha)$ , that failed to induce a sale. From property 2 of Lemma 3,  $b(u, S; \alpha) > S$ . As such, the distribution of buyer valuations is truncated from above at  $b(u, S; \alpha)$  and has positive density at all values in  $[v, b(u, S; \alpha)]$ . The probability that the auction in period  $t + 1$  ends in a price of at least  $S$  is equal to the probability that  $X_1$  exceeds the screening level in period  $t + 1$  and that both  $X_1$  and  $X_2$  exceed  $S$ . This occurs with

positive probability by the fact that  $b(u, S; a) > S$ . This establishes that there exists some  $\bar{\tau} \geq 1$ , such that

$$P\{p_{t+\tau} \geq S | S = s_t, \eta_{t+\tau-1} = 1\} > 0.$$

## A.8 Proof of Proposition 5

For a period- $t$  starting price of  $S = \sigma(\beta_t; 1)$ ,

$$G^1(S; t, \tau) = P\{X_2 \geq S, X_1 \geq \max\{\beta_{t+\tau}, S\} | X_1 < \beta_{t+\tau-1}\} - P\{X_1 \geq \beta_t\}.$$

For a period- $t$  reserve price of  $R = \sigma(\beta_t; 0)$ ,

$$\begin{aligned} G^0(R; t, \tau) &= P\{X_2 \geq \max\{\beta_{t+\tau}, R\} | X_1 < \beta_{t+\tau-1}\} - P\{X_1 \geq \beta_t\} \\ &= P\{X_2 \geq \max\{\beta_{t+\tau}, R\}, X_1 \geq \beta_{t+\tau} | X_1 < \beta_{t+\tau-1}\} - P\{X_1 \geq \beta_t\}. \end{aligned}$$

It is enough to show that  $\max\{\beta_{t+\tau}, R\} \geq R > S$ , which was shown by property 3 of Lemma 3.

## A.9 Proof of Proposition 6

Consider the equilibrium of a game with an arbitrary number of periods, with the number of periods denoted  $T$ . Conditional upon period  $T$  being reached, in which the starting price is reduced to  $\underline{v}$ , the highest possible valuation type must be below  $\beta_{T-1}$ . Conditional upon the state being  $\beta_{T-1}$ , the probability that the period- $T$  auction ends with a

price exceeding  $\sigma(\beta_t; 1)$ , the period- $t$  starting price, is

$$\begin{aligned} & P\{price \geq S | S = \sigma(\beta_t; 1), \eta_{T-1} = 1\} \\ &= \int_{\sigma(\beta_t; 1)}^{\beta_{T-1}} \int_{\sigma(\beta_t; 1)}^{x_1} f_{X_1, X_2}(x_1, x_2) dx_2 dx_1 / F(\beta_{T-1})^n, \end{aligned} \quad (28)$$

where  $f_{X_1, X_2}$  is the joint distribution of  $\langle X_1, X_2 \rangle$ . In period  $t$ , the probability that the auction ends in excess of  $\sigma(\beta_t; 1)$  is

$$P\{price > S | S = \sigma(\beta_t; 1)\} = \int_{\beta_1}^{\bar{v}} \int_{s_1}^{x_1} f_{X_1, X_2}(x_1, x_2) dx_2 dx_1. \quad (29)$$

$G^1(S; 1, 1)$  is the difference between (28) and (29).

As  $\delta \rightarrow 1$ , from Proposition 2  $\sigma(\beta_t; 1) \rightarrow \underline{v}$ . Thus for a given sequence of screening levels, as  $\delta \rightarrow 1$ , the numerator in (28) goes to  $F(\beta_{T-1})^n$ , so that the probability that the price in period  $T$  exceeds  $\sigma(\beta_t; 1)$  goes to 1. However, as  $\delta \rightarrow 1$ , the term in (29) goes to  $1 - F(\beta_1)^n$ , which is less than 1 for any  $\beta_1 > \underline{v}$ .

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# CHAPTER 2

## NON-STANDARD PREFERENCES AND IRRATIONAL BEHAVIOR IN AUCTIONS

### **1 Introduction**

A large literature has emerged over the last two decades that indicates that many bidders in auctions exhibit irrational behavior and non-standard preferences. A common finding is that some buyers overbid relative to contemporaneous auctions for similar items and fixed-price alternatives and consequently pay too much. A general lesson that can be inferred from this literature is that the standard rational model has shortcomings that are serious enough to warrant a richer approach to modeling bidder behavior.

While incorporating insights from psychology and related fields into traditional models of preferences has been shown to be important in many economic settings (e.g., DellaVigna 2009), the current study finds that bidders in auctions behave in ways that appear more consistent with standard rational behavior than some recent evidence might suggest. Using data we collected in a field experiment on eBay and supplemented with observational data from eBay, we test a range of non-standard and irrational behaviors that have been attributed to bidders and do not find evidence that they are important in a field setting.

We start by providing a benchmark theoretical model of what we will consider as standard rational bidder behavior. The model captures many of the important elements of the eBay auction environment for the product we examine – new DVDs for popular movie titles. The framework involves simultaneous auctions for identical items that differ only in their starting prices. The model follows closely the model in Peters and Severinov (2006) (“P&S” hereafter), with the main difference being to include non-zero

and heterogeneous information acquisition costs that potential buyers incur to identify and track the auctions in which the items of interest are being sold (P&S assume a frictionless environment). Given that many items on eBay including popular DVD movie titles are often sold in dozens or even hundreds of non-homogenous auctions running simultaneously, we believe that many buyers may reasonably not be aware of all relevant auctions and their current characteristics. Thus we expect these costs, which may result from factors such as differing facilities with internet use, values of time, and disutility of searching for and monitoring competing auctions, to affect how closely the bidder comes to obtaining the lowest available price.

An important prediction of the model is that ending prices across the two auctions are equal in expectation.<sup>1</sup> Overbidding relative to contemporaneous auctions can occur since some buyers are not aware of the alternatives. However, under the assumption that bidders are equally likely to find low versus high starting price auctions during their search, the incidence of overbidding is insensitive to starting price. In contrast, for reasons we describe below, the behavioral theories that we consider, predict that ending prices are decreasing in starting prices, in expectation, and that overbidding is more likely to occur in auctions with lower starting prices. These differing predictions regarding the effect of starting price offer a simple way to distinguish between these two sets of theories.<sup>2</sup>

We consider the following behavioral mechanisms. First is irrational herding, proposed in Simonsohn and Ariely (2008) (“S&A” hereafter), where bidders herd into auctions with a greater number of existing bids even though existing bids provide no valuable information. The implication is that winners overpay relative to contemporaneous auctions with fewer bids.<sup>3</sup> Second are opponent effects, which encompass a range of

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<sup>1</sup>This result is subject to certain conditions that we expand upon in Section II. P&S provide a stronger result that ending prices equate exactly.

<sup>2</sup>This starting price prediction is not new. For example, it provides a central test in Simonsohn and Ariely (2008). However, the results we present are new.

<sup>3</sup>This mechanism stands in contrast to rational herding (Banerjee 1992; Bikhchandani, Hirshleifer, and



behaviors in which bidders respond to opponents in ways that represent non-standard preferences. Opponent effects include spite, in which a bidder receives disutility from the surplus of opponents; joy of winning, in which the bidder receives utility from winning independent of her valuation for the item; or competitive arousal, in which the bidder is caught up in the heat of the moment and overbids. All of these behaviors cause overbidding relative to the bidders' initial valuations.<sup>4</sup> Third is the quasi-endowment effect, proposed in Heyman, Orhun, and Ariely (2004), where active bidders develop a sense of ownership over the item even before the auction has closed. In the spirit of the endowment effect (Thaler 1980), this sense of ownership causes bidders to bid beyond their initial valuations. A related fourth behavior is escalation of commitment, proposed by Ku, Galinsky, and Murnighan (2006), where bidders continue to participate in an auction beyond their valuations in order to justify the sunk costs of the time they have already committed to the auction.

While all of these behavioral theories differ as to the mechanism proposed, they offer a common prediction: A low starting price facilitates bidding activity at low prices, and this activity itself becomes a trigger for heightened bidding activity at higher prices. This trigger then leads auctions with lower starting prices to: (i) receive further bids at higher prices; (ii) close with higher ending prices; and (iii) result in a higher incidence of overbidding.

To test the competing predictions provided under standard and non-standard preferences, we conducted a field experiment on eBay in which we sold 210 matched pairs of movie-DVDs where the two DVDs in each pair were auctioned simultaneously from the same seller (us) but in separate auctions, one with a low starting price (\$0.99) and one

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Welch 1992), whereby the observed behavior of previous movers reveals valuable information in settings of incomplete information, and can imply that herding is rational.

<sup>4</sup>Kagel, Harstad, and Levin (1987) and Kagel and Levin (1993) are early papers that show that subjects in lab experiments overbid in second-price auctions. Morgan, Steiglitz, and Reis (2003), Heyman, Orhun, and Ariely (2004), Ku, Malhotra, and Murnighan (2005), Ku, Galinsky, and Murnighan (2006) and others provide more recent evidence in support of the individual mechanisms.

with a high starting price (average starting price of \$6.85).<sup>5</sup> This simple treatment generates exogenous variation in starting prices that allows us to estimate the causal effect of starting price on bidding outcomes. Exogeneity is important to ensure that the starting price is not being chosen in response to expected demand.<sup>6</sup> For example, since the starting price effectively functions as a reserve price, a seller may adjust starting price in response to expected demand. If starting price is endogenous to unobserved demand, the estimated effect of starting price may capture variation in demand, and hence variation in bidding outcomes that are correlated with but not caused by starting price itself. The matched-pairs aspect of the experiment allows us to include a fixed effect for each matched pair to control for time-varying unobserved demand for the particular movie title, thereby helping to isolate the causal effect of starting price.

We conduct three sets of tests based on this starting price variation. First, we investigate the relationship between starting price and ending price directly. We find that the average ending price of the low starting price auction is not statistically different than the average ending price of the matched high starting price auction conditional on both auctions exceeding the starting price in the high starting price auction. We also find that conditional on the high starting price auction failing to result in a sale, the average ending price of the low starting price auction is less than the starting price of the high starting price auction. These results support the standard model and are inconsistent with the behavioral mechanisms.

Next, we identify the 96 individual incidences of overbidding among the 210 matched

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<sup>5</sup>Auctions on eBay typically follow the dynamic second price auction (“DSPA”) format. In a DSPA, buyers are free to bid as many times as they choose and at any time they choose during a predetermined window set by the seller. At any point during the auction, buyers can observe the standing price, which is equal to: (i) the starting price if one or fewer bids have been received; or (ii) the second-highest bid plus a small increment if two or more bids have been received. Buyers also observe the number of bids at any point during the auction. The standing price is updated in real time until the time runs out. The winner is the high bidder when the time runs out and pays a price equal to the standing price at that point in time.

<sup>6</sup>Variations in demand in any particular auction may arise due to a range of factors; for example, differences in the number of contemporaneous competing auctions for the same DVD or the amount of time since the movie title was initially released on DVD.

pairs, and test directly for patterns of behavior consistent with each of the behavioral mechanisms.<sup>7</sup> Again we find little evidence of these behaviors.<sup>8</sup> Evidence of overbidding appears to be consistent with heterogeneous search costs, whereby a subset of buyers are aware of only one auction within the pair and bid rationally in that auction, given their limited awareness of alternatives.

Finally we investigate the effect of starting price on the likelihood that an auction receives an additional bid. Since auctions with lower starting prices have a greater number of bids on average upon reaching a given standing price than would an auction with a higher starting price, under irrational herding, subsequent bidders are more likely to bid in the auction with the lower starting price. Further, since auctions using lower starting prices tend to receive bids earlier, the high bidder in an auction started at a lower price is more likely to have been the high bidder longer than the high bidder in an auction with a higher starting price. It follows that under the quasi-endowment and escalation of commitment effects that the high bidder in the low starting price auction is more likely to increase his bid in response to being outbid. Lastly, to the extent that increased activity at lower prices can increase the intensity of motives such as spite, joy of winning or competitive arousal, so too do opponent effects predict greater bidding in lower starting price auctions. Given its consistency across the different mechanisms, we test the hypothesis that, controlling for standing price, an auction with a lower starting price is more likely to receive an additional bid.

The results of our test using experimental data indicate that starting price has no effect on the probability that the auction receives additional bids, providing further evidence that the behavioral mechanisms are not important among eBay participants. To demonstrate that our experimental results are not an artifact of our experimental pro-

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<sup>7</sup>Overbidding is identified when the winning prices of one of the paired auctions exceeds the other by at least one bid increment (fifty cents).

<sup>8</sup>Section V presents more specific tests of non-standard behavior using evidence of overbidding across matched pairs.

cedures, we replicate our analysis using observational data we collected from eBay and find very similar results (the collected data are similar to data used by previous empirical studies).

The study that is closest to ours is Lee and Malmendier (2010) (“L&M” hereafter), who also find that overbidding is common, and that opponent, quasi-endowment and escalation of commitment effects are unlikely to be the cause. Given that the two studies use different datasets and methodologies but arrive at the same conclusions, we believe the evidence against these behavioral mechanisms in the eBay setting is strong. Regarding explaining the overbidding that is observed, we offer an alternative explanation than L&M: While they argue that some bidders irrationally overlook lower-priced fixed-price options, and hence exhibit irrational limited attention, we believe that overbidding could naturally be explained within a standard rational model such as ours.

A key difference in interpretation between the two studies is whether all bidders are aware of all listings upon entering an auction. L&M assert that all bidders can be expected to have identified most or all listings for the desired item but sometimes overlook the more attractive options. We believe the task of identifying the most attractive options is likely to be quite costly in terms of time and effort given the large amount of information to search through. Once we allow some bidders to be unaware of some options, it may no longer be irrational for the bidder not to switch to a lower-price competing fixed-price option after entering an auction, or to enter the auction with a higher expected price than a fixed price option in the first place.<sup>9</sup> We also do not find support for the irrational herding empirical findings in S&A, which conduct tests very similar to those in the current study. A possible explanation for the difference in findings is that our estimation strategy allows us to more precisely control for time-varying unob-

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<sup>9</sup>Note also that the focuses of the two papers differ somewhat. L&M investigate bidder behavior with respect to fixed-price alternatives while we focus on bidder behavior regarding simultaneous competing auctions, and hence we also explore issues specific to competing auctions such as cross-bidding and herding behavior.

served demand, and hence more accurately isolate the direct effect of starting price on ending price. We discuss our results in relation to these previous studies in more detail in Section VII.

The current results have implications for the literature on overbidding in auctions conducted in the laboratory and speak to the question of the applicability of laboratory findings to the field setting (List 2006, Levitt and List 2007 and Levitt and List 2008). Furthermore, List (2003) shows that certain irrational biases are reduced with market experience, and Cooper and Fang (2008) show that laboratory subjects learn through experience to avoid behaviors that lead to overbidding and speculate that those who do not are weeded out with time. These previous findings may help to explain the differences between previous laboratory results and our findings from the field.

The rest of the article proceeds as follows: Section II discusses the testable implications of our theoretical model of rational behavior under standard preferences and distinguishes these predictions from those of the behavioral models; Section III discusses the experimental and observational data, Section IV describes the results of our test regarding the effect of starting price on ending price, Section V presents more specific tests of non-standard behavior using evidence of overbidding across matched pairs, Section VI describes results regarding the effect of starting price on the probability of the auction receiving additional bids, Section VII discusses the applicability of our results to previous studies and Section VIII concludes.

## **2 Theoretical Predictions**

We derive empirical predictions from a model of simultaneous auctions under two alternative sets of assumptions. Under the first, buyers are perfectly rational, though some buyers are imperfectly informed about one of the auctions. Under the second, buyers do not have standard rational preferences but instead earn additional utility from win-

ning an auction with a lower starting price. The added utility from winning an auction with a lower starting price captures the empirical predictions of the behavioral theories of irrational herding, opponent effects, quasi-endowment effect, and escalation of commitment while abstracting away from the underlying mechanism that generates this premium.<sup>10</sup> We provide the two models in the following subsections, and highlight the distinguishing testable predictions.

## 2.1 Standard Rational Model with Unaware Buyers

The following provides the underlying framework for analyzing the outcomes of our matched-pairs field experiment under the assumption that buyers are rational. Consider two auctions,  $L$  and  $H$ , selling identical items simultaneously. The auctions, differ only in their starting prices denoted  $S_L$  and  $S_H$  respectively, where  $S_L \leq S_H$ . The demand side of the market consists of  $m \geq 2$  buyers, each with unit demand for the item. Buyers' valuations, denoted  $v$ , are private information, independent and identically distributed according to some distribution  $F$ , assumed to have full support over the grid  $\Omega = \{S_L, S_L + d, S_L + 2d, \dots, v_{max}\}$ , for some step size  $d > 0$ . A buyer with valuation  $v$  who obtains a single unit at price  $p$  receives surplus  $v - p$ .

The model as described thus far, as well as the description of the auction mechanism employed herein, follows closely to that of P&S. In P&S, buyers move sequentially, having the opportunity to bid any amount in any available auction when it is their turn to bid.<sup>11</sup> The bidding continues up to a point at which all bidders decline to place additional bids. P&S show that there exists a perfect-Bayesian equilibrium of the game in which buyers follow a strategy in which each bid is one step above the standing price in the auction with the lowest standing price. P&S call this an efficient strategy as it allocates

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<sup>10</sup>Lee and Malmedier (2010) employ a similar utility-of-winning modeling assumption.

<sup>11</sup>P&S model an arbitrary number of auctions. We simplify their analysis in restricting attention to just two auctions to provide a closer analogy to our experimental setup.

the items to the participants with the highest valuations. Furthermore, this strategy leads to all winning bidders paying the same price.

Thus, the efficient strategy of P&S supports an outcome under rational bidding consistent with the Law of One Price. A limitation of the efficient equilibrium in P&S is that it does not allow for ending prices to differ across auctions in a given pair and hence does not allow us to understand the observed instances of overbidding. To allow for variation in prices, we introduce the concept of “unaware bidding,” whereby some subset of buyers only considers bidding in one of the two auctions within a pair.

Unaware bidding can be attributable to rational considerations such as search costs, which could lead to certain buyers not being aware of the second auction, choosing to forgo additional search when the expected gain from seeking out all of the contemporaneous alternatives is smaller than the time cost of additional search. Though our experiment was designed so as to minimize the effort of finding and bidding in the paired auction, time constraints may still prevent rational buyers from implementing the efficient strategy of P&S.<sup>12</sup> This behavior serves as a rational counterpart to the irrational “limited attention,” which L&M use to explain their results. Whatever the root cause of unaware bidding, we assume that unaware buyers bid rationally given they are aware of only one of the two auctions in the pair. We incorporate unaware bidding into the model by assuming that with some probability, each of the  $m$  buyers is aware of only one of the two auctions in a pair, and is equally likely to be aware of auction L versus auction H. In this way, some subset of buyers will only bid in one auction, irrespective of the bidding in the competing auction.

The game proceeds as follows. First, the starting prices of the two auctions are announced simultaneously. Buyers then arrive sequentially. Upon arriving, a buyer is given an opportunity to bid in either auction. A bid may be any amount on the grid,  $\Omega$ ,

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<sup>12</sup>For example, since the listings of our paired auctions were identical except for the starting price, the auctions would appear next to each other when search results are sorted by time remaining in the auction or most relevant results, but may appear far from each other when sorted by current standing price.

at least some minimum increment,  $e > 0$ , greater than the standing price in the chosen auction, if the auction has received at least one bid, or at least as high as the auction's starting price, otherwise. After receiving a bid, the standing price and identity of the high bidder of the auction are updated.<sup>13</sup> After the buyer that has entered most recently finishes submitting bid(s), each buyer that had entered earlier is given the opportunity, in order of his or her entry, either to submit new bid(s) (in either auction) or to pass. Once each buyer in the market chooses to pass, a new buyer can enter. After all  $m$  buyers have entered, the bidding process continues as bidders update their bids one after another. The order of bidding at this stage is the same as the order of entry: after the last buyer to enter submits bid(s), the first buyer to enter is called upon next, followed by the second buyer and so on. Bidding continues until all buyers pass. Then the high bidder in each auction obtains the item at a price equal to the auction's standing price as of the last bid. This final standing price is the ending price.

Having specified the model, we now establish a perfect-Bayesian equilibrium in which aware buyers bid up prices of the two auctions incrementally while unaware buyers bid only in the auction they are aware of. The bidding strategies are formalized as follows.

**Definition 1** *The bidding strategy of aware buyers is  $\alpha^*$ , defined as:*

- (a) *If the buyer is the current high bidder in either auction, or if the buyer's valuation is less than or equal to the lowest standing price, the buyer passes.*

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<sup>13</sup>Additional details that correspond to eBay exactly are: In determining the standing price, the second-highest bid in an auction refers to the second-highest bid received by a distinct bidder and if two or more bidders submit the same high bid, the first submitter is the high bidder. A condition in P&S and our model that differs from the eBay setting but greatly simplifies the analysis is that the standing price is equal to the current second-highest bid when at least two bids have been placed. On eBay, the standing price when at least two bids have been placed is the second-highest bid plus the minimum bid increment,  $e$ , which is fifty cents for most of the relevant range. This simplification is equivalent to assuming that bidders only consider bid increases of at least one dollar, perhaps because bidders incur a small time/effort cost to placing a bid that makes it not worthwhile to bid in very small increments. We expect the same (or very similar) predictions would hold if this incrementing were accounted for precisely.



**(b)** *Otherwise, if one auction has a lower standing price, the buyer bids in this auction.*

*If both auctions have the same standing price, the buyer bids in the auction in which it is more likely that the high bid equals the standing price, where the inference over the high bid is derived from Bayes' rule accounting for the equilibrium strategies. If both auctions have the same standing price and the high bids are equally likely to equal the starting price, the buyer bids in one of the two auctions with equal probability. The bid amount is the smallest value on the grid above the standing price.*

Note that strategy  $\alpha^*$  is the efficient strategy in P&S for the two-auction setting. While part (a) of the definition of  $\alpha^*$  is straightforward, the rationale for part (b) requires further explanation. Upon choosing between two auctions that have the same standing price, a buyer prefers to bid in the auction in which she is more likely to become the high bidder. Bidding in accordance with part (b) requires the buyer to identify the auction that gives her the greatest chance of becoming a high bidder.

**Definition 2** *The bidding strategy of unaware buyers is  $\beta^*$ , defined as:*

**(a)** *If the buyer is the current high bidder or if the buyer's valuation is less than or equal to the standing price in the auction the buyer is participating in, the buyer passes.*

**(b)** *Otherwise, the buyer bids an amount equal to his valuation in the auction he is participating in.*

Note that we use female pronouns to indicate aware buyers and male pronouns to indicate unaware buyers. Since unaware buyers participate in one auction only, the game from their perspective resembles a Vickrey auction (Vickrey 1961 ). In such a game, any strategy that has the buyer obtain the item at a price up to but no higher than his valuation is weakly dominant;  $\beta^*$  is one such strategy. However, given that aware

buyers make inferences about the magnitude of the high bid, which in turn governs their bidding decisions, the game is not equivalent to a Vickrey auction. Specifically, an aware buyer bids in auction  $j$ , conditional upon both auctions having the same standing price, if she believes that the high bidder in auction  $j$  is more likely to be an aware buyer. Bidding in accordance with  $\beta^*$  in auction  $j$  minimizes the probability that an aware buyer bids in  $j$  when the two auctions have the same standing price.<sup>14</sup>

**Proposition 1** *It is a perfect-Bayesian equilibrium for aware buyers to use strategy  $\alpha^*$  and for unaware buyers to use strategy  $\beta^*$ .*

All proofs are contained in the Appendix. In deriving our empirical predictions of the model, let  $P_H$  and  $P_L$  denote the standing prices in auction  $L$  and  $H$  respectively upon the close of the auction. As such, if auction  $j$  results in a sale,  $P_j$  denotes its ending price, while if auction  $j$  fails to sell,  $P_j$  denotes its starting price,  $S_j$ . Further, let  $E$  represent the expectation operator.

Corollaries 1 and 2 provide empirical predictions regarding expected ending prices and incidences of price divergence. As we will see in the next subsection, each of these predictions has a behavioral counterpart that distinguishes the two models.

**Corollary 1** *Expected prices. For a given matched pair of auctions in the equilibrium characterized by  $\alpha^*$  and  $\beta^*$ :*

1. *Conditional upon  $P_j \geq S_H + e$  for each  $j \in \{L, H\}$ ,  $E[P_H] = E[P_L]$ ;*
2. *Conditional upon auction  $H$  receiving zero bids,  $E[P_L] < S_H$ .*

In understanding the logic behind Corollary 1, it is important to recognize that the difference in starting prices across the two auctions serves as an exogenous source of variation, which gives rise to increased bidding activity in auction  $L$  at lower standing

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<sup>14</sup>For further explanation of this result, see the proof of Theorem 1 in the Appendix.

prices. This increased bidding activity in auction  $L$  leads to distinct predictions under the rational and behavioral models. At the same time, the difference in starting prices can differentially affect the ending prices, even in the absence of the behavioral mechanisms, by requiring that the ending price in auction  $H$  be at least  $S_H$ . Therefore, in order to isolate the effect of the behavioral mechanisms relative to the standard rational model, it is necessary to restrict attention to pairs in which the presence of the high starting price does not bind on the ending price. By restricting attention to pairs in which both auctions have ending prices of at least  $S_H + e$ , the comparison of ending prices amounts to a comparison of the second-highest valuation of all buyers in each auction. Under the standard rational model, the expectation of the second-highest valuation in auction  $L$  is equal to that of auction  $H$ .

Our next set of results involves price divergence, which we define as follows.

**Definition 3** *For a given matched pair of auctions,  $j$  and  $-j$ , we say that auction  $j$  is a divergent auction if:*

- (a)  $P_j \geq S_H + e$ ; and
- (b)  $P_j - P_{-j} > e$ .

**Corollary 2** *Price divergence. For a given matched pair of auctions in the equilibrium characterized by  $\alpha^*$  and  $\beta^*$ , a divergent auction is equally likely to be auction  $H$  as auction  $L$ .*

The rationale behind Corollary 2 is similar to that of Corollary 1. The presence of the higher starting price in auction  $H$  makes a higher ending price more likely in the absence of any behavioral mechanisms: whereas auction  $L$  requires two bidders to bid at least  $S_H$  to achieve a price of  $S_H$ , auction  $H$  requires only one. Therefore, by requiring the divergent auction to have an ending price strictly greater than  $S_H$ , insures that the

ending price in the divergent auction is determined by the second-highest valuation of all buyers in that auction. In that case, the divergent auction is the auction in which the second-highest buyer valuation is higher. Under the standard rational model, the divergent auction is equally likely to be auction  $L$  as it is to be auction  $H$ .

## 2.2 Utility of Winning the Low Starting Price Auction

To incorporate irrational bidding/non-standard preferences into the simultaneous auction game, we assume that buyers receive additional utility  $\pi > 0$  upon obtaining the item in the auction with the lower standing price. The inclusion of the premium,  $\pi$ , captures heuristically the added incentive present in each of the behavioral theories for buyers to obtain the item in auction  $L$ , while abstracting away from the specific mechanisms under which the premium is formed. For instance, under irrational herding, the premium arises when a new entrant observes one auction with a greater number of distinct bidders. Since auction  $L$  is more likely to receive bids from a greater number of distinct bidders, controlling for standing price, the premium represents the average difference in standing prices that would make a new entrant indifferent between bidding in the two auctions. Under quasi-endowment, the premium arises when a buyer becomes the high bidder and increases in the amount of time she holds this position. Since a high bidder in auction  $L$  will have been high bidder for longer on average than a high bidder in auction  $H$ , the premium is the average difference in standing prices such that upon being outbid, a buyer would be indifferent between which of the two auctions to place her next bid. The same rationale can be applied to opponent effects and escalation of commitment.

In analyzing the model, the premium,  $\pi$ , simply shifts each buyer's payoffs in a manner that is equivalent to raising the standing price in auction  $H$  by  $\pi$  under standard preferences. The equilibrium of the game resembles the efficient equilibrium of P&S,

subject to buyers perceiving the standing price in auction  $H$  to be  $\pi$  greater than its actual amount.

**Definition 4** *For a given matched pair of auctions,  $L$  and  $H$ , a  $\pi$ -shifted set of standing prices is the actual standing price for auction  $L$ , and the actual standing price plus  $\pi$  for auction  $H$ .*

**Definition 5** *The bidding strategy for all buyers is  $\alpha^\pi$ , defined as:*

- (a) *If the buyer is the current high bidder in any auction, or if the buyer's valuation is less than or equal to the lowest standing price, the buyer passes.*
- (b) *Otherwise, if one auction has a lower  $\pi$ -shifted standing price, the buyer bids in this auction. If both auctions have the same  $\pi$ -shifted standing price and there is one auction in which either the standing price has changed since the last change of the high bidder or that has yet to receive a bid, the buyer bids in this auction. Otherwise, the buyer bids in one of the two auctions with equal probability. The bid amount is the smallest value on the grid above the standing price.*

**Proposition 2** *It is a perfect-Bayesian equilibrium for all buyers to use strategy  $\alpha^\pi$  in the game in which buyers earn a premium of  $\pi$  in obtaining an item in auction  $L$ .*

Having established an equilibrium of the game in which buyers exhibit irrational behavior of non-standard preferences, we derive empirical predictions to contrast with those of the standard rational model.

**Corollary 3** *Expected prices. For a given matched pair of auctions in the equilibrium characterized by  $\alpha^\pi$ :*

1. *Conditional upon  $P_j \geq S_H + e$  for each  $j \in \{L, H\}$ , then  $P_L = P_H + \pi$ ;*

2. Conditional upon auction  $H$  receiving zero bids, then  $E[P_L]$  may be greater than or less than  $S_H$ .

**Corollary 4** *Price divergence.* Assuming  $\pi > e$ , then for a given matched pair of auctions in the equilibrium characterized by  $\alpha^\pi$ , a divergent price auction is more likely to be auction  $L$  than it is to be auction  $H$ .

Corollaries 3 and 4 are the behavioral analogues to Corollaries 1 and 2. Whereas Corollary 1 predicts that the ending prices in the two auctions will be equal in expectation, Corollary 3 predicts that auction  $L$  will have a higher ending price than auction  $H$ , not just in expectation, but in a point-wise manner since all buyers are now assumed to be aware. Further, since auction  $L$  has a higher ending price than auction  $H$ , auction  $L$  will be a divergent auction.

### 3 Data

While much of the literature surveyed in the introduction relies on observational data, we seek to test for irrational bidding or non-standard preferences using data from of a field experiment. Experimental data provide two advantages over observational data. First, by analyzing only auctions for which we have set the starting price, we can ensure that starting price is not set in response to expected demand, which we might expect sellers to do, and which may occur in ways that are unobserved in the data. Second, the matched-pairs feature of the experimental data allow us to compare outcomes across matched pairs, thus controlling for unobserved heterogeneity in a way that is only imperfectly replicable in observational data.

In what follows, we explain how the field experiment was performed and summarize the data. This explanation is followed by a description of the collection of observational data.

### 3.1 Experimental Procedures and Data

The data consist of the starting prices, ending prices, and bid histories of 420 auctions of new movie-DVDs run on eBay from July 13 to August 22, 2007. The experiment was designed as follows: Twenty-one DVD titles were chosen from Billboard magazine's best-seller list for June 2007. We excluded all television series and special edition sets from the original list of 25, leaving us with only feature-length films. Each title was auctioned in pairs, simultaneously, one with a starting price of 99¢ and the other with some higher price, chosen uniquely for that pair. All 21 pairs, 42 items in total, were then auctioned simultaneously as a cohort. The entire experiment consisted of 10 such non-overlapping cohorts for a total of 210 pairs of auctions and 420 total auctions.

The functioning of each auction and the appearance of each auction page was standardized to the greatest extent possible. Each auction in a cohort was started on the same date and time of day and set to end exactly 72 hours after the start. Each item was listed with a fixed shipping charge of \$3.00. The layout of the auction webpage was kept uniform across auctions. The product description was identical within each title and differed across titles only by the name of the film and by DVD format (widescreen or full screen).<sup>15</sup> To avoid any perception of product heterogeneity, the product description clearly mentioned that the DVD for sale was new and in its original shrink wrap.<sup>16</sup>

The starting price chosen for the high-starting-price treatment,  $S_{ic}$ , was chosen distinctly for each title  $i$  in cohort  $c$ . Since average prices were expected to differ across titles in a cohort, and also over time for a given title, the  $S_{ic}$  were chosen distinctly so as to provide our test with sufficient power. We arrived at each  $S_{ic}$  by applying a small increment to the average selling price for that title reported by eBay the day the auction

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<sup>15</sup>Figure 2.2 in the Appendix provides an example of the layout of the auction page and the product description.

<sup>16</sup>As it is customary on eBay for buyers of multiple items to receive bulk discounts on shipping, the item description explicitly stated that bulk discounts would not be given.

was listed.<sup>17</sup> In the first five cohorts, the increment was 10%, in the latter five, 25%. The change in markup had little effect on the probability of sale or the average price.<sup>18</sup>

Table 2.1 provides a summary of the average outcomes across the two starting-price treatments. Of the 210 auctions performed under the low-starting-price treatment (“LSPAs”), all of them sold and for an average ending price of \$6.88. Of those auctions performed under the high-starting price treatment (“HSPAs”), starting prices ranged from \$3.75 to \$11.25 with a mean of \$6.85. Within HSPAs, 81.9% of the items sold. Conditional upon a sale taking place, the average ending price across HSPAs was \$7.76. That this amount exceeds \$6.88, the average across LSPAs, is consistent with the predictions of auction theory based on standard preferences due to the truncation of auctions that would have ended in a lower price in the absence of a high starting price.<sup>19</sup> As expected, the LSPAs received more bids and were bid upon by more bidders than were HSPAs. LSPAs averaged 7.5 bids and 4.7 bidders on average as opposed to 1.6 and 1.4 respectively for HSPAs. Commensurate with having been bid upon by more bidders, LSPA listings were viewed by more prospective bidders, 30.4 on average, than were HSPA listings, an average of 16.7.<sup>20</sup>

Table 2.1 further shows that the median rating of winners in HSPAs and LSPAs respectively. An eBay participant’s rating is acquired through completed transactions

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<sup>17</sup>When listing an item for sale, eBay displays the average selling price for that title over the previous week. In determining the high starting price, we added an increment to this amount. Our use of this approach is motivated by recognizing that certain titles tend to sell at higher prices than others. By basing the high starting price on this value, we ensure that the starting price is high enough such that the price fails to be reached with sufficiently high probability to provide meaningful comparisons across the two auctions within the pair. Given that our analysis relies only on comparisons of matched pairs, any possible endogeneity problem due to our choice of starting price would be alleviated.

<sup>18</sup>In cohorts 1-5, 87 of 105 high starting price auctions resulted in sale; in cohorts 6-10, 85 of 105 high starting price auctions resulted in a sale. While average prices declined in cohorts 6-10, due to the fact that a portion of market demand had already been satisfied, the average difference between end prices of paired low and high starting price auctions remained a constant 37 cents across the two groups of cohorts.

<sup>19</sup>That the unconditional average ending price across HSPAs, \$6.36, was below that of LSPAs, does not allow us to infer anything about buyer behavior beyond the fact that the chosen high starting prices were not the revenue maximizing values.

<sup>20</sup>Each auction run on eBay has a counter that tracks the number of distinct identities that click on the auction’s listing to access the auction’s unique URL.



on eBay. Sellers can evaluate buyers with a positive (+1), negative (-1) or neutral score (0); the buyer's rating is the sum of all of these scores. With a median rating of 85, the winners of HSPAs in our data were slightly more experienced than winners of LSPAs who had a median rating of 78. However, given the standard deviation in bidder rating across unique bidders of 462, the difference is not statistically significant.

Table 2.1: Summary statistics by starting price treatment

	LSPA	HSPA
Number of auctions	210	210
Number of auctions resulting in sale	210	172
Average starting price	\$0.99	\$6.85
Average ending price conditional on sale	\$6.88	\$7.76
Average ending price unconditional on sale	\$6.88	\$6.36
Average number of bidders unconditional on sale	4.7	1.4
Average number of bids unconditional on sale	7.5	1.6
Average number of views unconditional on sale	30.4	16.7
Median rating of high bidder	78.0	85.5

## 3.2 Observational Data

To collect observational data from eBay, we created a Java programming tool that makes use of API handles available on eBay for most aspects of each auction. We collected bid-level data on all auctions for DVDs of the bestselling movie titles in August and September 2008 according to Billboard magazine that sold between September and November 2008.

For each auction, the data record: item characteristics, including condition (five categories from Acceptable to Brand New), format (DVD, Blu-ray, HD-DVD), title, whether the item was relisted, whether the listing had enhancements such as bold letters, shipping cost and method (e.g., first-class, media), and whether the DVD is a special edition,

widescreen, or unrated version; seller characteristics such as the net number of positive feedback and percent of feedback that are positive, their location, whether their seller ID has changed, and whether they have an eBay store or are a power seller; and bid characteristics, including amount and time of each bid, whether the bid is an automatic proxy bid or actual bid, and an identifier that allows us to determine whether they are a first-time or repeat bidder in the auction.<sup>21,22</sup> From the bid data, we construct each standing price reached throughout the course of the auction, and an indicator variable indicating whether an additional bid is placed. A change in the standing price is prompted by the placing of actual bids, so there is no loss in restricting our attention to only actual bids. In total, our working dataset consists of 9,095 bids (excluding proxy bids) from 1,733 auctions and 1,037 unique sellers covering sixteen movie titles.

The observational data is used to corroborate our results from testing the effect of starting price on the probability of receiving bids. The results of the next two sections rely on the experimental data only.

## 4 Starting Price versus Ending Price

We now test the first part of the uniform price prediction of the standard model, given in Corollary 1, which is that the low and high starting price auctions have the same ending price on average, conditional on receiving at least two bids. The alternative is the behavioral prediction, given in Corollary 3, which is that low starting price auctions have a higher ending price on average than high starting price auctions, conditional on

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<sup>21</sup>The amount of the actual bid itself is available for all non-winning bids.

<sup>22</sup>The proxy bidding system employed by eBay works as follows: when a bid is placed by someone other than the current high bidder, the current high bidder's bid is automatically increased up to his actual bid amount if the new bid exceeds his bid, or up to the new bid plus the minimum increment if the new bid is less than his actual bid. A bid placed by the high bidder has no effect on the standing price, but increases the maximum amount that the proxy bidding system will bid for him. In what follows we distinguish between *proxy bids*, which is the bid automatically placed up to the buyer's maximum, and *actual bids*, which is the amount entered into the system by the bidder, which serves as the maximum proxy bid.

both auctions receiving at least two bids.

Table 2.2 presents the average ending prices for LSPAs and HSPAs across all 51 auctions pairs (out of 210) in which both auctions ended with a price of at least  $S_H + e$ . The average ending price for HSPAs is 40 cents larger than that for LSPAs. A t-test of the null hypothesis that the average ending prices are the same fails to reject the null, and has a p-value of 0.14.<sup>23</sup> When the true difference is 10 percent of the average ending prices of the 51 HSPAs (86 cents), the power of the test to reject the null hypothesis at a 5 percent significance level is 88 percent.<sup>24</sup>

Table 2.2: Test of average ending prices of completed auctions

	Average ending price	Standard error	N	t-statistic	p-value
LSPA	8.16	0.24	51		
HSPA	8.56	0.34	51		
Difference	-0.40	0.27	51	-1.48	0.14
<i>After removing outliers:</i>					
Difference	-0.14	0.20	49	-0.69	0.49

We next test the second part of the uniform price predictions of the standard rational model, given in Corollary 1, which is that the ending price of low starting price auctions is below the starting price of high starting price auctions on average conditional on the high starting price auction not resulting in sale. The alternative is the behavioral prediction, given in Corollary 3, which is that the average ending price of LSPAs may be above or below the starting price of the HSPA, conditional on the HSPA not resulting

<sup>23</sup>All p-values throughout the paper are calculated using a two-tailed test.

<sup>24</sup>The price differences in the two outliers, -\$8.00 and -\$5.32 respectively, represent the only two values such that there was not a corresponding price difference as large in the opposite direction.

in sale. The test is weaker than the test above since the two sets of predictions are only distinguishable if the average ending price of the LSPAs is above the average starting price of the HSPAs.

Table 2.3 reports the average ending prices of the 38 LSPAs where the corresponding HSPA failed to sell. The average ending price of the LSPAs is \$1.15 less than the average starting price of the paired HSPA. A t-test rejects the null hypothesis that the amounts are equal with a p-value less than .01. The results of these two tests are consistent with the standard rational model, and the result of the first test is strongly inconsistent with the behavioral predictions.

Table 2.3: Test of average prices when HSPA received no bids

	Average price (end- ing/starting)	Standard error	N	t-statistic	p-value
LSPA	5.77	0.34	38		
HSPA	6.92	0.32	38		
Difference	-1.15	0.27	38	-4.27	<0.0001

The predictions in Section II do not address paired auctions in which the LSPA and HSPA both sell but the HSPA receives only one bid. If these auctions were sufficiently prevalent, one may wonder whether this more complete set of auctions (104 pairs of auctions in total) would show evidence of the behavioral mechanisms. Table 2.4 shows that the average ending price of this broader set of LSPAs is 6 cents higher than the average ending price of the corresponding HSPAs. With a p-value of 0.74, a t-test that this amount is statistically different than zero fails by a wide margin to reject the null hypothesis. In comparison to the test presented in Table 2.2, this test biases the result in favor of LSPAs as it includes pairs in which the ending price in the HSPA exactly

equals  $S_H$ . Thus, the failure of this test to find higher ending prices amongst LSPAs is condemning to the existence of the purported behavioral mechanisms.

Table 2.4: Expanded test of average ending prices of completed auctions

	Average ending price	Standard error	N	t-statistic	p-value
LSPA	7.93	0.17	104		
HSPA	7.88	0.11	104		
Difference	0.06	0.18	104	0.33	0.74
<i>After removing outlier:</i>					
Difference	0.14	0.16	103	0.85	0.40

## 5 Individual Cases of Overbidding

The results of the previous section showed that on average, ending prices are consistent with standard rational model and inconsistent with behavioral mechanisms. However, we still observe in the data many individual instances of overbidding; that is, instances where the winning bidder in one auction paid more than the winning bidder in the paired auction. In this section, we document the patterns of overbidding that occur in the experimental data, and then examine the individual cases for evidence of the behavioral mechanisms.

To document the patterns of overbidding in individual matched pairs, we employ the concept of price divergence that was introduced in Section II. Consideration of divergent auctions restricts attention to only those pairs in which the auction with the higher ending price receives at least two bids equal to or greater than  $S_H + 0.5$ . This indicates that the second-highest valuation buyer present in this auction has a valuation of at least

$S_H + 0.5$ . When it can be inferred that the second-highest valuation of buyers across the two auctions differ by more than 50 cents, then we say that the auction with the higher ending price is a divergent auction. Since we are not concerned with the magnitude of the observed price differences, a feature of this analysis is that we can incorporate a much larger sample than in the analysis of the previous section.

Table 2.5 groups individual cases of price divergence by whether the divergent auction is a HSPA (column “HSPA”) or a LSPA (column “LSPA”). We see that among divergent auctions, HSPAs slightly outnumber LSPAs 50 to 46.<sup>25</sup> These groups are further subdivided as to whether the ending price in the paired auction,  $P_{-j}$  ended with a price of at least  $S_H + e$  or not. We observe similar patterns whether or not we include pairs in which the lower ending-price auction’s ending price is below  $S_H + e$ . In what follows, we further examine individual cases of overbidding for evidence of the behavioral mechanisms.

Table 2.5: Divergent price auctions by starting price treatment and by ending price in

	paired auction		
	LSPA	HSPA	TOTAL
$P_{-j} \geq S_H + e$	14	19	33
$P_{-j} < S_H + e$	32	31	63
TOTAL	46	50	96

## 5.1 Irrational Herding

To test the extent to which irrational herding can explain the incidence of divergent auctions, we focus on auctions won by first-time bidders. Low starting prices correspond to the presence of more bidders at any given standing price; hence, under irrational

<sup>25</sup>Of the 210 sample pairs, those not included in this table are: 18 pairs in which either both auctions resulted in a sale and their ending prices were within 50 cents of one another and 96 pairs in which no comparison of ending prices could be made.

herding, buyers that arrive later observe more bidders in the LSPA, and consequently enter the LSPA with a higher probability. The causal mechanism is illustrated below:

*Lower starting price → More bidders enter at lower prices, attracting more bidders at higher prices → Higher ending price*

Our test of irrational herding proceeds as follows:

1. Demonstrate the link between a lower starting price and the entry of bidders, substantiating the first causal arrow above; then
2. Show that divergent auctions won by first time bidders are not more likely to have been LSPAs, negating the second causal arrow above.

Consistent with the first causal arrow, winning bidders in LSPAs were in fact more likely to have placed their initial bid in the auction with a greater number of existing bidders than were winning bidders in HSPAs. The percentage of winning bidders in LSPAs who placed their first bid in the LSPA when that auction had, at the time, more competing bidders than the paired HSPA, was 87.6%. In comparison, there were no winning bidders in HSPAs who placed their first bid in the HSPA when that auction had, at the time, more competing bidders than the LSPA.<sup>26</sup>

To demonstrate an effect on divergent auctions caused by irrational herding, it must then be the case that – consistent with the second causal arrow above – winners of LSPAs are also more likely to be divergent bidders. This is the prediction of Corollary 4. We find this not to be the case as only 36 out of the 78 divergent price auctions won by first-time bidders were LSPAs. We test for statistical significance using a binomial test, the results of which are presented in Table 2.6. Recall from Corollary 2 that under the standard rational model, we expect divergent auctions to be equally LSPAs as HSPAs.

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<sup>26</sup>This difference in proportions is statistically significant at a level of significance less than one in ten thousand.

Therefore, we calculate the p-value based on a two tailed test, where under the null hypothesis, the Bernoulli probability of a divergent auction won by a first-time bidder being a LSPA is 0.5. The column labeled “# Trials” indicates the total number of divergent auctions for auctions won by first-time bidders and repeat bidders respectively, whereas the column labeled “# Successes” indicates the number of those that are LSPAs. Given a p-value of 0.57, we fail to find evidence that divergent auctions are more likely to be LSPAs.

Table 2.6: Results of binomial tests

	# Successes	# Trials	p-value
First-time bidders	36	78	0.57
Repeat bidders	9	17	0.63

## 5.2 Quasi-endowment and Opponent Effects

We next examine repeat bidders to test the extent to which quasi-endowment and opponent effects can explain overbidding. Since low starting prices lead to bidders entering the auction earlier, the bidders will spend more time on average as the high bidder. To the extent that a bidder becomes more attached to the item as she spends more time as the high-bidder, the quasi-endowment effect predicts that the bidder should be willing to pay more for the item. Escalation of commitment (Ku, Galinsky, and Murnighan 2006) works similarly. Since low starting prices lead bidders to participate in the auction for a longer period of time, the sunk cost of participating is on average higher for lower starting prices. To the extent that sunk costs generate a desire to justify these costs by winning the item, escalation of commitment will be more evident in LSPAs. Likewise, since lower starting prices lead to more bidders at a given standing price (most importantly to multiple bidders participating as opposed to one bidder), opponent effects causing repeat bidders to overbid are more likely to appear in LSPAs. The causal mechanism is illustrated below:



*Lower starting price  $\rightarrow$  Earlier bidding at lower prices, greater attachment  
at higher prices  $\rightarrow$  Higher ending price*

Our test of quasi-endowment and opponent effects proceeds as follows:

1. Demonstrate the link between a lower starting price and the entry of bidders, substantiating the first causal arrow above; then
2. Show that divergent auctions won by first time bidders are not more likely to have been LSPAs, negating the second causal arrow above.

The applicability of the results we present in this section to escalation of commitment and to opponent effects is subject to the implication that lower starting prices are in fact correlated with increased opponent effects and feelings of attachment and commitment. These assertions are difficult to support empirically. However, we can demonstrate that low starting prices do indeed correspond to winning bidders having spent more time as the high bidder (a necessary condition for attachment), and low starting prices correspond to winning bidders having spent more time in the auction overall, measured as the amount of time remaining in the auction upon entering (a necessary condition for commitment and opponent effects).

Supporting our approach to testing for the quasi-endowment effect, we find that winners of LSPAs spent longer as high bidder than winners of HSPAs. As the theory of the quasi-endowment effect suggests that divergent bidding becomes more likely the more attached the potential buyer is to the object, we use the amount of time the buyer spent as high bidder as a proxy for attachment. We look for winning bidders whose final bid was a repeat bid and separate them as to whether they had previously been high bidder for over  $E$  hours. The parameter,  $E$ , is varied, as it is not evident to us the length of time required for a person to become attached to an item. Figure 2.1 shows the number of auctions won by repeat bidders across values of  $E$  further distinguished by

whether the auction was a LSPA or HSPA. Consistent with the first causal arrow above, for each value of  $E$  considered, winning bidders in LSPAs, whose winning bid was a repeat bid, were more likely than those in HSPAs to have previously been the high bidder for at least  $E$  hours at some point during the auction.<sup>27</sup> Thus, if the quasi-endowment effect can be said to explain the incidence of divergent auctions, it must also be the case that divergent auctions won by repeat bidders are more likely to be LSPAs. This is the prediction given by Corollary 4.

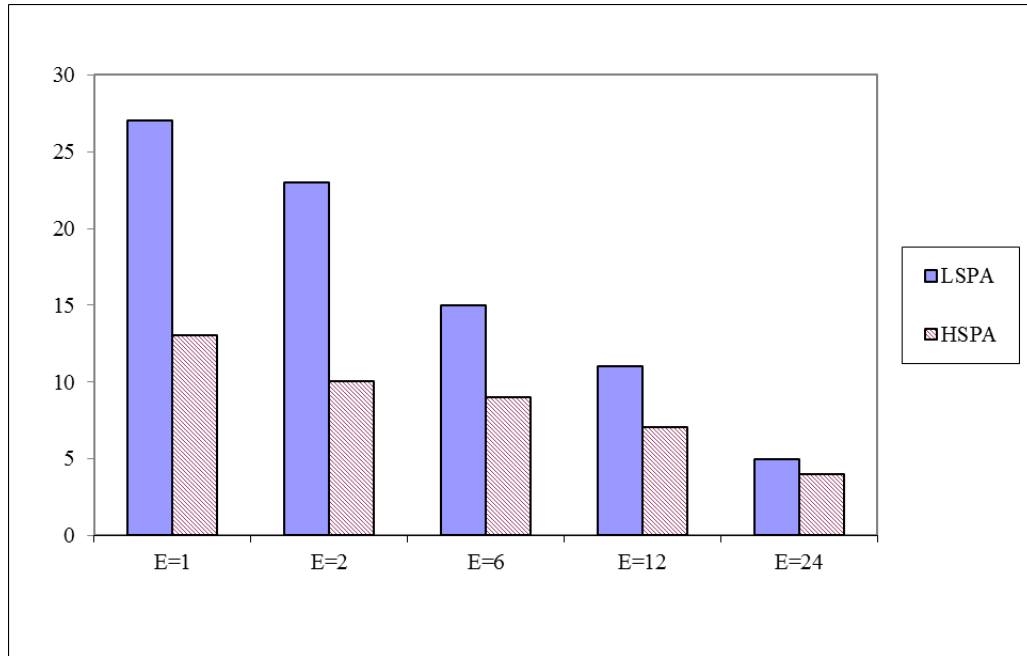


Figure 2.1: Alternate measures of quasi-endowment by starting price treatment

We find that contrary to the predictions of the behavioral mechanisms, divergent auctions won by repeat bidders are not more likely to be LSPAs. Table 2.6 presents the results of our analysis, showing that nine of the 17 divergent auctions won by repeat bidders were LSPAs. The p-value of 0.63, which was calculated in a similar fashion to our analysis of first-time bidders, does not provide sufficient evidence to reject the null hypothesis and instead provides evidence consistent with the standard rational model.

<sup>27</sup>These differences for  $E$  equal to 1 and 2 are statistically significant at  $\alpha < 0.05$ . At higher values of  $E$ , the discrepancy between winners of LSPAs and of HSPAs decreases somewhat; coupled with the smaller sample size, this renders the differences not significant even at the  $\alpha = 0.10$  level.

Our two analyses of divergent auctions show that auctions won by a first-time bidder or a repeat bidder are not more likely to be LSPAs. As such, we do not find evidence of any theory that predicts divergent auctions by a mechanism predicated upon a lower starting price, including irrational herding, opponent effects, quasi-endowment effect or escalation of commitment.

## 6 Starting Price versus the Probability of Additional Bids

We now examine whether, conditional on the current standing price, auctions with lower starting prices are more likely to receive additional bids. Irrational herding predicts that since auctions started at lower prices attract more bidders at lower standing prices, they should also attract more bidders at higher ones, thus making the receipt of an additional bid more likely. Similarly, opponent effects and quasi-endowment effects predict that the increased bidding activity that occurs at lower prices in LSPAs can intensify motives such as spite, joy of winning, and competitive arousal, and increase the buyers sense of ownership over the item, which make the buyer more likely to overbid in response to being outbid.

### 6.1 Baseline Model

In performing our test of additional bids, the unit of observation is the individual bid. We wish to estimate the effect of starting price on the probability that the auction receives an additional bid conditional on the current standing price of the auction.<sup>28</sup> Consider an auction  $j$  that has reached  $k = 1, \dots, K$  distinct standing prices within the sample.<sup>29</sup>

<sup>28</sup>S&A conduct a very similar test, which is discussed in the following section.

<sup>29</sup>Since an auction would have received one bid before entering the sample, the  $K$  distinct standing prices correspond to  $K + 1$  distinct bids. Auctions that have yet to receive a bid have been excluded from the sample as the starting price in such an auction is not comparable to a standing price in an auction that has received bids. For instance, starting prices in auctions that have not been bid upon may reflect a non-serious or unrealistic strategy by the seller.

For each auction  $j = 1, \dots, N$ ,  $X_j$  are auction-specific characteristics that remain constant across observations within a given auction, which include movie title, seller rating, end date, duration of the auction, and starting price.  $Z_{jk}$  are observation-specific characteristics that vary by bid, which include standing price, time left in the auction, and the number of distinct bidders that have placed bids to that point. The dependent variable  $y_{jk} = 1$  if the auction receives an additional bid, and  $y_{jk} = 0$  otherwise. The structural equation is,

$$y_{jk} = I\{X_j\beta + Z_{jk}\delta + U_{jk} > 0\}, \quad (1)$$

where  $U_{jk}$  a mean-zero random error.

## 6.2 Unobserved Demand

Unobserved demand for the item at the time of an auction is an important but complicating determinant of whether an auction receives an additional bid. There will be unobserved demand for the particular movie title that varies by the particular date and time of day. For example, demand for a Batman movie may be highest after the title is released and during the evenings when more buyers are online. However, demand for any particular auction at that time may depend on the number of contemporaneous auctions for the same title. We exploit the matched-pairs aspect of our experimental design by including a matched-pair fixed effect to control for any time-varying unobserved demand for that particular movie title.

Formally, let,  $\gamma_g$  denote the realization of latent demand for observations in group  $g$  (the matched pairs). We can express the outcome of observation  $k$  in auction  $j$  and group  $g$  as:

$$y_{gjk} = I\{X_j\gamma + Z_{jk}\delta + \gamma_g + U_{gjk} > 0\}. \quad (2)$$

The above formulation implicitly assumes that all potential buyers are aware of all contemporaneous auctions in that all auctions within a given group will be equally affected by unobserved demand. However, in the presence of unaware buyers, there may still be important variation in unobserved demand across the two auctions. To understand how this unobserved demand may operate, first consider a HSPA with starting price of  $S$  that has received one bid. The standing price will also be equal to  $S$  (recall that eBay employs a second-price auction), and only one bidder with a valuation of at least  $S$  must be present for this observation to appear in the data. Next suppose the LSPA has been bid up to the standing price  $S$ . For this auction, at least two bidders with valuations of at least  $S$  must be present for the observation to appear in the data. Note that either auction will receive an additional bid if a buyer is present who is not currently the high bidder but has a valuation above  $S$ . However, the presence of the second bidder in the LSPA whose valuation is at least  $S$  indicates a higher expected unobserved demand in the LSPA versus the HSPA, for which only one bidder has demonstrated a valuation of at least  $S$ . This higher expected unobserved demand in the LSPA indicates a higher probability of an additional bid in the LSPA conditional on the standing price  $S$ , particularly from an unaware buyer. Without properly accounting for unobserved demand of this form, the estimates will attribute a higher probability of an additional bid to the low starting price of the LSPA, when in fact the higher probability of an additional bid is due to higher unobserved demand. Further, we expect this type of unobserved demand to have a greater effect on our observational data than the experimental data since the observational data consist of more than two concurrent auctions, thus making it more difficult (and hence more costly) for a buyer to be aware of the standing prices in all concurrent auctions.

To address this second type of demand, we remove observations for which  $k = 1$  from our estimation of equation (2). All observations with a standing price of  $S$  and for which  $k > 1$  are such that we can infer from their inclusion in the data that there are at

least two potential buyers for the item with valuations of at least  $S$ .

Table 2.7: Effect of starting price on probability of additional bid – experimental data

	(1)	(2)	(3)	(4)
Starting price	0.887 (0.029)***	0.959 (0.038)	0.966 (0.042)	1.091 (0.072)
Log of minutes remaining	1.375 (0.048)***	1.421 (0.082)***	1.455 (0.059)***	1.867 (0.194)***
Log of seller rating	0.852 (0.081)*		0.815 (0.090)*	
Conditional on matched pair?	No	Yes	No	Yes
$k = 1$ observations excluded?	No	No	Yes	Yes
Observations	1892	1896	1494	1496

### 6.3 Experimental Results

The results of our estimation using experimental data are presented in Table 2.7, and are estimated using logit and conditional models with estimates reported as odds ratios. The model in column 1 is based on equation (1) and indicates that the marginal effect of starting price on the dependent variable is negative (a coefficient less than one in the logit specification) and is statistically significant at the one percent level. Column 3 presents the estimated model with the  $k = 1$  observations removed. The effect of starting price on the dependent variable is significantly reduced relative to column 1 and is not-statistically distinguishable from zero (reported as a coefficient value indistinguishable from one). Column 2 estimates equation (2) whereby a group is defined as the LSPA and HSPA within a given matched pair. Again, the effect of starting price is significantly reduced relative to column 1 and is not-statistically significant. Column 4 combines the regressions reported in columns 2 and 3 respectively by grouping by

matched pairs and excluding the  $k = 1$  observations. The effect of starting price is again statistically indistinguishable from zero.

The estimates indicate that after controlling for unobserved heterogeneity, starting price has little or no effect on the probability that an auction receives an additional bid. The contrast in results between the estimate of the starting price effect in column 1 and in columns 2-4 highlights the importance of controlling for unobserved heterogeneity, which evidently appears to predict starting price and also the probability of additional bids.<sup>30</sup>

## 6.4 Observational Results

One may wonder whether the comparison of two identical items auctioned by the same seller but with different starting prices is too obvious even for a buyer who would behave irrationally in a more ambiguous setting. In that light, we now corroborate the results of the previous section using the observational data.

We now examine data consisting of bidding outcomes at all standing prices reached within all auctions on eBay with at least one bid that met our inclusion criteria (title, date, auction format, etc.). We wish to employ the group fixed effects approach as specified by equation (2). Without the matched pairs feature of the field experiment, we can only imperfectly control for latent demand, which we do by including group fixed effects where the group is defined to be DVD title, item quality (new versus used), and auction end date. The logic for this grouping is the same as that used in the matched pairs analysis: that the pool of buyers present for one auction within a group is similar or the same as the pool of buyers for the other auctions in that group. Nevertheless, unobserved demand may also be reflected in the  $k = 1$  versus  $k > 1$  distinction.

The results of our estimation are presented in Table 2.8. Columns 1 and 3 present

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<sup>30</sup>Similar results are obtained under a probit specification (which is the specification employed by S&A). This is discussed further in the following section.

the results of our estimation of equation (1) with the  $k = 1$  observations included and excluded respectively. That the odds ratios on starting price are less than 1 and statistically significant at the 1% level, indicates that starting price is inversely related to the dependent variable. Column 2 presents the odds ratios from a conditional logit regression, which includes group effects but includes the  $k = 1$  observations. Again, the odds ratio on starting price is less than 1 and statistically significant at the 1% level. Column 4 presents the result of the conditional logit using only the  $k > 1$  observations. The odds ratio on starting price is not-statistically different from zero, indicating no effect of starting price on the probability of receiving a bid.

Table 2.8: Effect of starting price on probability of additional bid – observational data

	(1)	(2)	(3)	(4)
Starting price	0.817 (0.025)***	0.858 (0.033)***	0.904 (0.029)***	1.002 (0.049)
Log of minutes remaining	1.459 (0.025)***	1.595 (0.039)***	1.493 (0.027)***	1.612 (0.048)***
Shipping charge	0.895 (0.100)	0.903 (0.109)	0.946 (0.058)	0.967 (0.075)
New dummy	1.239 (0.135)**		1.186 (0.147)	
Log of seller rating	1.010 (0.021)	1.033 (0.031)	1.001 (0.023)	1.044 (0.042)
Seller positive percent rating	1.052 (0.023)**	1.050 (0.032)	1.042 (0.026)*	1.085 (0.022)***
Conditional on matched pair?	No	Yes	No	Yes
$k = 1$ observations excluded?	No	No	Yes	Yes
Observations	5706	5640	4420	4370



We explained in the introduction that the behavioral mechanisms predict a negative relationship between starting price and the incidence of further bidding while the standard model predicts no relationship. The results of this section are again consistent with a standard rational model and not the behavioral mechanisms.

## 7 Discussion

There are several results in two closely related studies, L&M and S&A, which are inconsistent with the current findings and warrant discussion. S&A conduct versions of the uniform price test and additional bid test that are presented in the current paper, yet find evidence that starting price is inversely related to ending prices and the probability of additional bids (whereas we find no relationship). We are able to replicate the findings in S&A for both the experimental and observational data by estimating regression models with similar specifications. Column 1 of Table 2.7 and column 1 of Table 2.8 contain these specifications, and both show an important role for starting price. However, when we more completely control for heterogeneity in unobserved demand across bids and auctions, the effect disappears. Column 4 of Table 2.7 and column 4 of Table 2.8 contain these results for the experimental and observational data.

This unobserved demand can be thought of as arising from a sample selection procedure whereby an observation only appears in the data if the auction received bids – and hence had high realizations of unobserved demand – at all previous standing prices. That is, unobserved demand is responsible for both the auction appearing in the data at the given standing price and also for the auction receiving a bid at that standing price. The surprising result that the probability of additional bids increases with standing price after controlling for item and seller characteristics in the results in column 1 of Table 2.7 and column 1 of Table 2.8 (in essence, that demand increases with price), and that the starting price-ending price relationship disappears after more completely controlling for

unobserved demand, is consistent with this selection mechanism.

While the outcomes of the current study are broadly consistent with those in L&M, we differ in interpreting the cause of the observed overbidding. L&M assert an irrational limited attention mechanism whereby buyers overlook plainly obvious but lower-priced alternatives to the auctions in which they are bidding. The key assumption is that all bidders know all available options upon entering the bidding, which makes it irrational to continue to bid in an auction once the standing price exceeds the fixed-price option, or, given the bidder faces switching costs of leaving the auction for the fixed-price option, makes it irrational for the bidder to enter the auction in the first place given that the auctions ending at a higher price on average than the fixed-price option. However, if bidders are not aware of all options, it may be rational to enter the auction and not to switch to a competing fixed-price option after entering.

To support the assumption that all bidders know all available options upon entering the bidding, L&M reconstruct for each bid in their data the set of all auctions and fixed prices available at the time of the bid, assuming bidders see only Cashflow 101 listings and sort the listings by remaining listing time. They then provide evidence that when the fixed-price option is farther away from the auction that is actually chosen, in terms of number of auctions listed between the chosen auction and the fixed-price option, the fixed-price option is more likely to be overlooked; and the further down on the screen the fixed-price option is listed, the more likely it is to be overlooked. We believe instead that eBay participants cannot be expected to learn all available options at reasonable cost.

We conducted an eBay search of the Cashflow 101 board game, which is the item of focus in the L&M study, using the search terms “Cashflow” versus “Cashflow 101,” and sorted the results by time remaining, ascending price, and descending price. The first ten search results for each search type are listed in Figure 2.3.<sup>31</sup> The first notable char-

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<sup>31</sup>See Appendix B. The left-most column indicates the ordering of the search results, the next column

acteristic of the search results is the large number of listings and differences in wording and product types the bidder faces: there are 66 listings for “Cashflow 101” and 245 listings for “Cashflow,” with audio versions, cassettes, CDs, Cashflow 101 versus 202, manuals, e-games, etc. Second it is apparent that which listings are seen by the bidder are sensitive to how the results are sorted.<sup>32</sup> Sorting by lowest price primarily returns auctions while sorting by highest price primarily returns fixed-price options (because the current price in an active auction is typically lower than the fixed-price alternative), and sorting by remaining-listing-time gives a mix of the two.

Wenyan and Bolivar (2008) of eBay! Research Labs report that 30 percent of eBay searches are sorted by a price ordering and not the remaining-listing-time default search ordering.<sup>33</sup> Thus, without knowing a bidder’s search terms and ordering, it appears infeasible to reconstruct with any precision the ordering of listings the bidder would have observed. Given the multitude and variety of listings, we believe that information acquisition costs are likely to affect bidder behavior, and that it is very plausible that the 17 percent of bidders that L&M show are responsible for the observed overbidding may have simply searched using different search terms, used different search orderings, and thus failed to account for all available items.<sup>34</sup>

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over provides the item’s search heading, and the next column over indicates whether the listing was for an auction, ‘A’, fixed price option, ‘B’, or a fixed price option with the option to bid, ‘A,B’. Listings in this last category are listed according to the fixed price, until a bid is received, after which the item is listed as an auction.

<sup>32</sup>The results of Figure 2.3 are re-sorted by lowest price first in Figure 2.4, then by highest price in Figure 2.5. See Appendix B.

<sup>33</sup>This statistic is for 2007 when remaining-listing-time was the default search ordering, as it was for the period studied by L&M. Starting in 2008, the default search ordering was changed to a new “best match” format.

<sup>34</sup>The findings of L&M that bidders are less likely to consider eBay items that are listed further away in the search results page from the auction that is actually chosen, and that this pattern is most pronounced at the time the bidder first enters an auction, is also consistent with information acquisition costs.

## 8 Conclusion

This paper takes a closer look at a range of non-standard preferences and irrational behaviors that have been proposed in recent years and finds little evidence that they are important in explaining auction outcomes in the field. Our conclusions are based on two sets of analyses: First we test the predictions of a theoretical model of rational buyers with standard preferences and heterogeneous search costs using experimental data. Second, we reassess a significant result reported in S&A by controlling for unobserved heterogeneity using both experimental and observational data.

In testing the predictions of our theoretical model, we begin by comparing average ending prices across the HSPA and LSPA treatments in the field experiment. The model of P&S assumes that buyers bid as many times as they please in either auction within the pair, which leads to the conclusion that when both auctions result in a sale, their ending prices will be equal. We consider a modification to P&S in which some subset of buyers is constrained to only bid in one auction, leading to the conclusion that conditional upon both the HSPA and the LSPA ending at prices strictly above the starting price in the HSPA, ending prices should be equal in expectation. This prediction stands in contrast to those of models of irrational bidding that predict the LSPA to end at a higher price. The results of our test fail to reject the null hypothesis that average ending prices are equal.

Next, we look at overbidding and test whether our theoretical model explains specific incidences of overbidding. In contrast to P&S, the model we present allows for variation in ending prices across auctions in a matched pair due to the presence of unaware buyers but predicts instances of overbidding as being equally likely to occur in a HSPA as a LSPA. This prediction stands in contrast to the behavioral models which predict that overbidding is more likely to occur in LSPAs. We find no evidence that overbidding is triggered by any of the behavioral mechanisms though they can be explained by unaware

bidding.

In the second set of analyses, we replicate the empirical strategy of S&A but more carefully control for unobserved demand, and find little evidence to support the prediction that lower starting prices lead to more bidding, conditional on standing price. Using the matched-pairs feature of the field experiment, we control for unobserved demand using a fixed effect for each pair. The effect of starting price on the probability of receiving a bid disappears under this specification, and separately under an alternative specification in which the  $k = 1$  observations are removed. With the observational data, we group auctions by DVD title/item quality/end date, which admittedly is a weaker control for unobserved demand since buyers may only enter an auction at the very end and may not be aware of other auctions ended earlier or ending later in the day. It is under this specification that controlling for  $k = 1$  versus  $k > 1$  becomes more important. The effect of starting price on the probability of receiving a bid disappears when this specification is coupled with the removal of the  $k = 1$  observations.<sup>35</sup>

In sum, the analyses we have performed all indicate that various behavioral mechanisms that have received a great deal of attention recently do not appear to be important in the eBay environment. Further, it appears that a more standard model in which buyers are rational but face search costs predict the outcomes of our experimental and observational data to a much greater extent. This paper, therefore, serves as a cautionary note that results that may appear unusual may have mundane explanations.

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<sup>35</sup>To more completely control for unobserved demand in the observational data, we suggest estimating the probability of the receipt of an additional bid using a sample selection framework in which the  $k$ th observation from a given auction only appears in the data, conditional upon the first  $k - 1$  observations receiving bids. This approach requires one to estimate a multivariate probit model, a computational burdensome exercise. This is a possible extension to the current paper, which we are currently pursuing and which will allow us to more explicitly control for unobserved demand and also run various counterfactual experiments using the resulting structural model.

# APPENDIX

## A Proofs

### A.1 Proof of Proposition 1

The proof proceeds in multiple parts, first establishing that there is no incentive for an aware buyer to deviate when all other aware buyers play according to  $\alpha^*$  and all unaware buyer play according to  $\beta^*$ ; next establishing that there is no incentive for an unaware buyer to deviate when all aware buyers play according to  $\alpha^*$  and all other unaware buyers play according to  $\beta^*$ . We spend considerably more time on the first. Lemmas 1 and 2 characterize the outcome following from some arbitrary state of the game in which all buyers play according to their proposed equilibrium strategies in what follows. Lemmas 3 - 5 establish that there is no incentive for an aware buyer to deviate from  $\alpha^*$  and Lemma 6 establishes that there is no incentive for an unaware buyer to deviate from  $\beta^*$ .

Let the state of the game be the array of buyers' valuations, sellers' standing prices together with the identities of buyers who have submitted them, the high bids together with the identities of the high bidders, the history of the standing and high bids, and the order in which the buyers move. There is a one-to-one relationship between the nodes in the game and its states. Precisely, the state of the game is a full description of the corresponding node in the game. Define a public state of the game as the union of all components of the state of the game that are publicly known. Specifically, the public state of the game includes the standing prices, the identities of the high bidders, the history of all these, and the order of moves. We will assign indices to buyers based on the order of their entry, with buyer 1 arriving first and having the first opportunity to submit new bid(s) after being outbid, and so on.

At each *information* set where a buyer is called to move, the buyer knows the public

state of the game, her own high bids and her bidding history. Information sets are partially ordered: one information set precedes the other if the latter can be reached via some sequence of moves from the former. A path of the game is a collection of all information sets such that for any pair of information sets in it one can be reached via some sequence of moves from the other. Since the profile of the standing bids in our bidding game is ascending, along any path, one information set precedes the other if and only if the standing bid at each seller at the former information set is (weakly) lower than at the latter.

The equilibrium requires that we specify beliefs for both aware and unaware buyers both on and off the equilibrium path.

**Definition 6** *The following characterize the beliefs of all buyers both on and off the equilibrium path:*

1. *Aware buyers know that with some probability  $\mu$ , each opposing bidder may be an unaware buyer who finds it too costly to discover the location of the paired auction.*
2. *Buyers do not know the aware-unaware status of other buyers and can only infer it based on each buyer's bidding.*
3. *Given equilibrium strategies  $\alpha^*$  and  $\beta^*$ , all buyers upon observing a buyer who has bid multiple times, but always in the auction with the lowest standing price, believe the buyer to be aware with probability one. The posterior beliefs about the buyer's high bid are updated as follows: if the standing price has not changed since the last change in high bidder, buyers infer that the high bid is at least greater than the standing price; if the standing price has changed once since the last change in high bidder, buyers infer that the high bid is equal to the standing price; if the standing price has changed more than once since the last change in high*

*bidder, then buyers' prior beliefs were incorrect as the bidding of the high bidder is inconsistent with  $\alpha^*$ .*

4. *Given  $\alpha^*$  and  $\beta^*$ , all buyers upon observing a buyer who has bid only once and the standing price has increased by at least  $2d$  since the buyer placed his bid, believe the buyer to be unaware with probability one. The posterior beliefs about the buyer's high bid are characterized by the posterior distribution of  $F$ , conditioning on the event that the bid is as large as the cutoff value inferred.*
5. *Given  $\alpha^*$  and  $\beta^*$ , all buyers upon observing a buyer bid only once, all buyers update their beliefs over whether the bidder is aware, given the likelihood that the observed bidding would be placed by an aware buyer and unaware buyer in accordance with  $\alpha^*$  and  $\beta^*$ , respectively. The posterior beliefs about the high bidders is a probability weighted distribution constructed by weighting the respective distributions conditional upon the buyer being aware and unaware by the probability that the buyer is aware and unaware.*
6. *If the observed bidding is inconsistent with  $\alpha^*$  and  $\beta^*$ , the posterior beliefs about the buyer's high bid are characterized by an arbitrary posterior distribution  $G$ , assigning positive probability over all values in  $\Omega$ , conditioning on the event that the bid is as large as the cutoff value inferred.*

Clearly, these beliefs are rational on an equilibrium path where all buyers follow  $\Lambda^* = \{\alpha^*, \beta^*\}$ .

The following establishes the notation used in characterizing the various states of the game. Let  $\Gamma$  denote some arbitrary state of the game and  $G(\Gamma)$  denote the continuation game starting from an information set corresponding to  $\Gamma$ . Consider then any point  $p$  on the grid, and let  $a_j(p; \Gamma) = 1$  if auction  $j$  has a standing price in state  $\Gamma$  of  $p$  or less, 0 otherwise. An auction for which  $a_j(p; \Gamma) = 0$  is one in which a bid of  $p$  or less would



not be possible in state  $\Gamma$  as either the starting price is above  $p$  or the standing price was bid up above  $p$  prior to state  $\Gamma$  being reached.

Turning to the demand side of the market, let  $b_i(p, j; \Gamma) = 1$ , for buyer  $i$ , a participant in auction  $j$ , that has a standing price of  $p$  in state  $\Gamma$ , if buyer  $i$  is not a high bidder in the paired auction and buyer  $i$ 's valuation is no less than  $p + d$ . A buyer for whom  $b_i(p, j; \Gamma) = 1$  is willing to bid in any state of the continuation game from state  $\Gamma$  where she does not hold a high bid in either auction and the standing price in auction  $j$  does not exceed  $p$  and is less than the standing price in the paired auction. Further, let  $b_i^d(p, j; \Gamma) = 1$ , if buyer  $i$  holds a high bid in auction  $j$  of at least  $p$  which was placed prior to state  $\Gamma$  and buyer  $i$ 's valuation is no greater than  $p$ , and equal to 0 otherwise. A buyer for whom  $b_i^d(p, j; \Gamma) = 1$  has already deviated from the equilibrium strategies and holds a high bid such that were she to be outbid, she would not bid further. Thus, even if she were to follow her prescribed equilibrium strategy going forward i.e. placing no further bids, she may still obtain an item at a price exceeding her valuation.

To denote the ending prices following state  $\Gamma$ , define  $P_j(\Gamma)$  such that,

$$P_j(\Gamma) = \max \{p \mid \sum_i [b_i(P_j(\Gamma) - d, j; \Gamma) + b_i^d(P_j(\Gamma) - d, j; \Gamma)] > 1, j \in \{L, H\}\}$$

if such a  $p$  exists,  $v_{\max}$  otherwise (recall that  $v_{\max}$  is the highest point on the grid). If  $a_j(P_j(\Gamma); \Gamma) = 1$ , then  $P_j(\Gamma)$  is the lowest standing price at which there is no incentive for buyers to bid further.

Given the above discussion, it will be useful to distinguish those outcomes, following stage  $\Gamma$ , in which ending prices across the two auctions coincide. In this way, let  $b_i^u(p, j; \Gamma) = 1$  for an unaware buyer  $i$ , who is active in auction  $j$  and whose valuation is no less than  $p + d$ , 0 otherwise. It should be noted that for an unaware buyer,  $b_i^u(p, j; \Gamma) = b_i(p, j; \Gamma)$ . Note further that  $b_i^u$  differs from  $b_i^d$  in that for  $b_i^d$  to equal 1 requires that the buyer deviated from the equilibrium strategy, whereas  $b_i^u$  equal to 1 is

consistent with the buyer playing in accordance with  $\beta^*$ .

**Lemma 1** *Upon an aware buyer  $i$  being called to bid in state  $\Gamma$ , when bidding in accordance with  $\alpha^*$ , the following describes the lowest standing price as of the time she passes:*

1. *If buyer  $i$  bids only in one auction  $j$ , then her last bid leaves auction  $j$  with a standing price no greater than the standing price in the paired auction.*
2. *If buyer  $i$  bids in both auctions, then if she stops bidding because the lowest standing price is no longer lower than her valuation, then after her last bid, the standing prices will be equal.*
3. *Otherwise, buyer  $i$  bids in both auctions and stops bidding upon becoming the high bidder in some auction  $j$ , in which case, the standing price in auction  $j$ , denoted  $p_j$ , will be either equal to or  $d$  less than the standing price in the paired auction. The standing prices will be equal after  $i$ 's last bid if upon reaching a state in which the standing prices in both auctions are  $p_j$ , she bids in auction  $j$ . Then this will be her last bid. If however, she bids in the paired auction,  $j'$ , whereby it has been assumed that the bid does not make her the high bidder, the standing price in  $j'$  will increase to  $p_j + d$ . Buyer  $i$ 's next and last bid will make her the high bidder in auction  $j$  with a standing price of  $p_j$ .*

**Proof.** Suppose first that when buyer  $i$  is called to bid, there is a unique lowest standing price in auction  $j$ , say. In accordance with  $\alpha^*$ , buyer  $i$  will bid in auction  $j$  in an amount  $d$  above the standing price. If the bid results in either buyer  $i$  becoming the high bidder in  $j$  or causes the standing price to increase to where the next highest allowable bid in either auction is above buyer  $i$ 's valuation, he will not bid again during this turn. Since the standing price in auction  $j$  will have increased by at most  $d$ , the fact that auction

$j$  had previously had a lower standing price implies that the current standing price in auction  $j$  can be no greater than the standing price in the paired auction.

Suppose then that the bid does not make  $i$  the high bidder in  $j$ , nor does it raise the standing price in  $j$  to  $i$ 's valuation, then  $i$  will continue to bid in  $j$  until the sooner of the following: (i) buyer  $i$  becomes the high bidder in  $j$ ; (ii) the standing price in  $j$  equals  $i$ 's valuation; or (iii) the standing price in  $j$  equals that of the paired auction. If i or ii occur first, then the standing price in auction  $j$  must be weakly lower than that of the paired auction, otherwise, iii would have occurred first. If iii does occur first, then in determining which auction to place her next bid, buyer  $i$  takes into account that the standing price in auction  $j$  has changed since the last change in high bidder. Therefore, if the standing price in the paired auction  $j'$  has not changed since the last change in high bidder, then buyer  $i$  bids in auction  $j$ . Otherwise,  $i$  must make inferences on the likelihood that the high price in each auction is equal to the standing price and bid accordingly. Regardless of which auction buyer  $i$  chooses, she will continue bidding, choosing amongst the two auctions in this manner until either becoming a high bidder in one auction or until the lowest standing price equals her valuation.

Suppose it is the latter. In that case, the standing prices in both auctions must equal buyer  $i$ 's valuation. If not, then buyer  $i$  must have bid in an auction an amount above her valuation, which is in violation of  $\alpha^*$ . Suppose instead that it is the former. Since buyer  $i$  bids in both auctions, then at some point prior to becoming high bidder, the standing prices in the two auctions must have been equal. Consider then the last such standing price at which the two auctions have the same standing price prior to buyer  $i$  becoming high bidder. There are then two cases to consider. First, suppose that buyer  $i$  becomes high bidder with his first bid placed in auction  $j$ , say. Since buyer  $i$  has become the high bidder, the standing price will not have changed, so the standing prices in the two auctions will remain equal. Next, suppose that buyer  $i$ 's bid in auction  $j$  does not make him the high bidder in that auction. Such a bid will necessarily raise the standing

price in auction  $j$ , by  $d$ . Buyer  $i$ 's next bid will then be in auction  $j'$ . If buyer  $i$ 's bid in  $j'$  makes him the high bidder, then the standing price in that auction will not have increased, thus leaving the standing price in  $j'$  to be  $d$  less than in auction  $j$ . Lastly, if buyer  $i$ 's bid in auction  $j'$  does not make him the high bidder in that auction, then his bid will have raised the standing price in auction  $j'$  to be equal to that of auction  $j$ . This contradicts the assertion that we were considering the last such standing price at which the two auctions have the same standing price prior to buyer  $i$  becoming a high bidder.

■

In what follows, let  $P^0(\Gamma) = \min_j \{P_j(\Gamma)\}$

**Lemma 2** *Consider any state  $\Gamma$ . If all aware and unaware buyers use  $\alpha^*$  and  $\beta^*$ , respectively, in  $G(\Gamma)$ , then the following must hold:*

1. *The ending price in any auction  $j$  that ends in a sale is  $P_j(\Gamma)$ .*
2. *Any auction  $j$  for which  $a_j(P_j(\Gamma); \Gamma) = 1$  results in a sale.*
3.  *$P_L(\Gamma) = P_H(\Gamma) = P^0(\Gamma)$  if either both auctions are won by aware buyers for whom  $b_i(P^0(\Gamma), j; \Gamma) = 1$  or if not, then if the second highest valuation in both auctions is that of an aware buyer for whom  $b_i(P^0(\Gamma) - d, j; \Gamma) = 1$ , in which case, the common ending price will be the aware buyer's valuation.*

**Proof.** The proof is by contradiction. Suppose that the ending price in auction  $j$  is some  $p' < P_j(\Gamma)$ . In that case,  $\sum_i b_i(p', j; \Gamma) > 1$ , so there exists a buyer  $i$  who is not a high bidder in either auction at a standing price strictly below his valuation. Therefore, upon being called upon to bid, buyer  $i$  would not pass, thus contradicting the fact that buyer  $i$  had bid in accordance with the prescribed equilibrium strategies.

Suppose then that the ending price in auction  $j$  is some  $p'' > P_j(\Gamma)$ . For the standing price in auction  $j$  to have reached  $p''$  implies the existence of a second bidder who had bid at least  $p''$ . By the construction of  $P_j(\Gamma)$ , such a bid must have come from a buyer

that was either a high bidder in the paired auction or whose valuation was strictly below  $p''$ , thus contradicting the assertion that all buyers bid in accordance with the prescribed equilibrium strategies.

To prove the second part, assume by way of contradiction that  $a_j(P_j(\Gamma); \Gamma) = 1$  and auction  $j$  does not receive any bids. By definition of  $P_j(\Gamma)$ , there is a buyer  $i$  whose valuation is at least  $P_j(\Gamma)$ , is not a high bidder in the paired auction, and who has not bid in auction  $j$ . When called to bid, buyer  $i$  would bid at least  $P_j(\Gamma)$  (if he is an aware buyer, he will  $P_j(\Gamma) + d$  in accordance with  $\alpha^*$ ; if he is unaware, he will bid his valuation in accordance with  $\beta^*$ , which is at least  $P_j(\Gamma) + d$ ), thus contradicting the assertion that all buyers had bid in accordance with the equilibrium strategies.

The third part of the lemma follows from Lemma 1. Since upon an aware buyer becoming the high bidder in an auction, the standing price in that auction can be no greater than the standing price in the paired auction, it follows that if both auctions have high bidders that are aware buyers, their standing prices must be equal.

Next, suppose this is not the case, but that the second highest valuation bidder in both auctions is an aware buyer. Upon an aware buyer  $i$  placing her last bid when not the high bidder, the standing prices in the two auctions must be equal. Therefore, it remains only to show that the result holds even when the aware buyer whose valuation determines the prices in both auctions is not the final bidder. It follows that the high bid in one of the auctions must have been placed after buyer  $i$  had become a high bidder, in auction  $j$ , say; if not then there will have been two bids placed since buyer  $i$ 's last bid in auction  $j$ , which would contradict the assertion that buyer  $i$ 's final bid determined the ending price in that auction. It follows that for buyer  $i$  not to have bid in response to being outbid in auction  $j$ , then the standing price upon being outbid must have equaled her valuation,  $v$ . Further, by Lemma 1, by the fact that buyer  $i$  had been a high bidder, the standing price in the paired auction,  $j'$ , must have been at least as high as in auction  $j$ . Specifically, since buyer  $i$ 's bid determines the price in auction  $j'$ , the standing price

in auction  $j'$  as of buyer  $i$  becoming the high bidder in auction  $j$  must have been  $v$ . ■

The first two parts of Lemma 2 provide what are akin to supply and demand arguments regarding the resulting ending prices in the two auctions respectively. The third part of the lemma indicates the conditions under which the ending prices in the two auctions will equate. Intuitively, this third result shows that as long at least one aware buyer is involved in determining the price in each auction (as either the high bidder or second-highest bidder), then their prices will equate. Thus, only if there exist at least two unaware buyers in an auction whose valuations exceed the highest bid placed by an aware buyer, will that auctions price diverge that of the paired auction. We focus on the highest bid, as opposed to the highest valuation, or an aware buyer as the highest valuation aware buyer need not bid up to her valuation in order to win the paired auction.

To formalize the previous argument, let  $Y_j^u$  denote the second-highest valuation of all unaware buyers in auction  $j$  and let  $Y_j$  denote the second highest value among the following: the valuations of all aware buyers; the valuations of all unaware buyers in auction  $j$ ; and auction  $j$ 's starting price. Then, in an equilibrium of the game in which buyers play in accordance with  $\alpha^*$  and  $\beta^*$  respectively, then if there exists a  $j$  such that  $Y_j^u \geq Y_{-j}$ , then the ending price in auction  $j$  exceeds that of  $-j$ .

The following three lemmas demonstrate that an aware buyer cannot profitably deviate from  $\alpha^*$  when all other aware buyers play in accordance with  $\alpha^*$  and all unaware buyers play in accordance with  $\beta^*$ . In what follows let  $M(P^0(\Gamma)) \equiv \{j | P_j(\Gamma) = P^0(\Gamma)\}$  and let  $g_i(p; \Gamma)$  denote the number of high bids help by buyer  $i$  in auctions with a standing price of at least  $p$  in  $G(\Gamma)$ .

**Lemma 3** *Suppose that in  $G(\Gamma)$ , all buyers other than  $i$  follow  $\Lambda^*$ , buyer  $i$  has valuation  $v_i$  and follows some strategy  $\alpha' \neq \alpha^*$  such that the lowest ending price is some  $P' < P^0(\Gamma)$ . Then for  $v_i \geq P^0(\Gamma)$ , in state  $\Gamma$ , buyer  $i$  does not obtain any units at a price strictly below  $P^0(\Gamma)$ .*

**Proof.** If  $v_i \geq P^0(\Gamma)$ , then by bidding in accordance with  $\alpha^*$ ,  $i$  obtains an item. By definition of  $P^0(\Gamma)$ , we have that for any auction  $j$  with an ending price such that  $P' < P^0(\Gamma)$ ,

$$\sum_i [b_i(P', j; \Gamma) + b_i^d(P', j; \Gamma)] > 1. \quad (3)$$

Therefore, there exists a buyer  $k \neq i$  such that  $b_k(P', j; \Gamma) + b_k^d(P', j; \Gamma) = 1$  who holds a high bid in  $j$  at a standing price of  $P'$ . Suppose not. If  $b_k(P', j; \Gamma) = 1$ , then since  $P'$  is the lowest standing price,  $k$  bids in  $j$  in accordance with  $\Lambda^*$ . If this bid fails to make  $k$  the high bidder in  $j$ , then it raises the standing price above  $P'$ , thus contradicting the assumption that  $P'$  is the lowest ending price. If instead,  $b_k^d(P', j; \Gamma) = 1$  and  $k$  is not the high bidder in  $j$ , then there must exist a second buyer with a bid in  $j$  of at least  $P'$ , thus contradicting the assumption that  $P'$  is the ending price in  $j$ .

Given the existence of such a  $k$  in every such  $j$ , it follows that for any  $P' \leq P^0(\Gamma) - d$ ,  $i$  cannot trade at any such  $j$  for a price less than  $P^0(\Gamma)$ . Lastly, since any other auction has a standing price of at least  $P^0(\Gamma)$ ,  $i$  cannot trade in those auctions for any price less than  $P^0(\Gamma)$ . ■

**Lemma 4** *Suppose that in  $G(\Gamma)$ , all buyer other than  $i$  follow  $\Lambda^*$ , buyer  $i$  follows some strategy  $\alpha' \neq \alpha^*$  such that the lowest ending price is  $P' > P^0(\Gamma)$ . Then the number of units obtained by  $i$  and her total payment is at least as large as when he follows  $\alpha^*$ .*

**Proof.** When bidding in accordance with  $\alpha^*$ , buyer  $i$  obtains at least  $I(\alpha^*; \Gamma)$  units where,

$$\begin{aligned} I(\alpha^*; \Gamma) &= \sum_{j \in M(P^0(\Gamma))} \sum_{k \neq i} [a_j(P^0(\Gamma); \Gamma) - b_k(P^0(\Gamma), j; \Gamma) - b_k^d(P^0(\Gamma), j; \Gamma)] \\ &\quad + \sum_{p=P^0(\Gamma)+d}^{v_{\max}} g_i(p; \Gamma). \end{aligned}$$

Upon deviating to  $\alpha'$  such that the lowest ending price is  $P' > P^0(\Gamma)$ ,  $i$  obtains at most

$I(\alpha'; \Gamma)$  units, where,

$$\begin{aligned} I(\alpha'; \Gamma) &= \sum_{j \in M(P')} \sum_{k \neq i} [a_j(P'; \Gamma) - b_k(P', j; \Gamma) - b_k^d(P', j; \Gamma)] \\ &\quad + \sum_{p=P'+d}^{v_{\max}} g_i(p; \Gamma). \end{aligned}$$

It follows that  $i$  purchases at least as many units playing  $\alpha'$  as with  $\alpha^*$  if:

$$I(\alpha'; \Gamma) \geq I(\alpha^*; \Gamma). \quad (4)$$

If  $P' = P^0(\Gamma)$ , inequality (4) holds trivially. Otherwise, let  $P' = P^0(\Gamma) + \eta d$ , for some  $\eta \in \mathbb{N}$ . Inequality (4) is implied by the following:

- $\sum_{j \in M(p+d)} a_j(p+d; \Gamma) - \sum_{j \in M(p)} a_j(p; \Gamma)$  is the number of auctions selling for exactly  $p+d$ .
- $[b_k(p+d, j; \Gamma) + b_k^d(p+d, j; \Gamma)] - [b_k(p, j; \Gamma) + b_k^d(p, j; \Gamma)]$  does not exceed the number of high bids that buyer  $j$  holds in  $\Gamma$  in auctions whose standing prices are  $p+d$ .
- $g_i(p+d; \Gamma)$  is the number of high bids that buyer  $i$  holds in  $\Gamma$  in auctions whose standing prices in  $\Gamma$  are exactly  $p+d$ .

■

**Lemma 5** *Suppose that in  $G(\Gamma)$ , all buyer other than  $i$  follow  $\Lambda^*$ , buyer  $i$  follows some strategy  $\alpha' \neq \alpha^*$  such that the lowest ending price is  $P^0(\Gamma)$ . Then  $i$ 's payoff is no higher than the payoff she obtains by following  $\alpha^*$ .*

**Proof.** If buyer  $i$  had not deviated from  $\alpha^*$  prior to state  $\Gamma$ , then when all other buyers play according to  $\Lambda^*$ , Lemmas 3 and 4 are sufficient to establish that there is no incentive to deviate in state  $\Gamma$ . Thus, the only buyers worth considering are those that have already



deviated prior to state  $\Gamma$ , either by becoming high bidder in two auctions or by becoming high bidder at a standing price above one's valuation.

Regardless of whether the deviation makes the bidder the high bidder in two auctions, deviation  $D_2$ , or at a standing price above her valuation, deviation  $D_1$ , bidding in accordance with  $\alpha^*$ , has buyer  $i$  pass until  $i$  is no longer the high bidder in an auction in which she would earn negative surplus would she to have won. If the buyer had committed deviation  $D_1$ , then  $\alpha^*$  has her pass for the duration of the auctions. If she had committed deviation  $D_2$ ,  $\alpha^*$  would call for her to bid if she were no longer a high bidder in either auction where the lowest standing price is below her valuation. Thus, following  $\alpha'$  in stage  $\Gamma$  has buyer  $i$  bid in either auction while still high bidder in an auction in which she earns negative surplus. This would necessarily consist of a bid in excess of one of her high bids were the buyer to have committed deviation  $D_2$ . If buyer  $i$  had committed deviation  $D_1$ , the deviation could also include a bid in the auction in which she is not the high bidder.

The only potential strategic purpose of a deviation bid in state  $\Gamma$  would be to influence the beliefs of other aware buyers to make them more likely to bid in the auction in which her surplus is lowest if she were to remain high bidder, thus supplanting buyer  $i$  as the high bidder. Given the definition of  $\alpha^*$ , such beliefs affect opposing aware buyers' bidding only upon reaching a state at which the two auctions have the same standing price. In such case, if the bidding of the high bidder in auction  $j$ , which is buyer  $i$ , was consistent with  $\alpha^*$  to that point and if the standing price had increased once since buyer  $i$  became the high bidder, then an opposing aware buyer  $i'$  would infer that the high bid in auction  $j$  was equal to its standing price. Thus, if the same were true in auction  $j'$ , buyer  $i'$  would randomize between the two auction. If the same inference could not be made for the high bid in auction  $j'$ , then buyer  $i'$  will bid in auction  $j$  with probability one and let buyer  $i$  off the hook. Note that buyer  $i$ 's beliefs regarding buyer  $i$ 's high bid matter only if the high bid in auction  $j'$  were equal to buyer  $i$ 's high bid. If the high bid

in auction  $j'$  were above buyer  $i$ 's high bid, then even if buyer  $i'$  bid in  $j'$  first, then upon failing to become the high bidder in  $j'$ , she would then bid in auction  $j$ .

The point to realize is that even if buyer  $i'$ , upon reaching state  $\Gamma$  in which she infers that the high bid of buyer  $i$  is equal to the standing price, any deviation from  $\alpha^*$  in  $\Gamma$  by buyer  $i$  would require buyer  $i'$  to revise his beliefs. Such a revision would have her believe that buyer  $i$ 's high bid were equal to or greater than the standing price, with an expectation determined by rule 5 in Definition 6. If, on the other hand, buyer  $i'$  already believed the high bid of buyer  $i$  to be determined by rule 5 of Definition 6, then any further deviation by buyer  $i$  will do nothing to change belief. Thus, we can infer that there is no deviation  $\alpha'$  that can positively affect the outcome of buyer  $i$ . Further, since any such deviation subjects  $i$  to the possibility of exacerbating her loss, by increasing the price she would pay or by causing her to become high bidder in a second auction, there is no incentive for  $i$  to deviate from  $\alpha^*$  in state  $\Gamma$ . ■

Having established that there is no incentive for an aware buyer to deviate from  $\alpha^*$  when all other buyers play according to  $\Lambda^*$ , we turn our attention to unaware buyers.

**Lemma 6** *When all buyers bid according to  $\Lambda^*$ , bidding according to  $\beta^*$  is a best response for an unaware buyer.*

**Proof.** Consider an unaware buyer  $i$ , with valuation  $v$ , who participates in auction  $j$  such if  $i$  follows  $\beta^*$  in stage  $\Gamma$ , the ending price in auction  $j$  is either  $P_j(\Gamma)$  or  $P_j(\Gamma) + d$ . First suppose that buyer  $i$  enters only after state  $\Gamma$ . Notice that if  $P_j(\Gamma) > v$ , then any deviation that has buyer  $i$  win the auction and pay a price no less than  $P_j(\Gamma)$  provides him with negative surplus. If  $P_j(\Gamma) + d \leq v$ , then a deviation that has buyer  $i$  lower his maximum bid from  $v$  can either have no effect on  $i$ 's surplus, or could cause him to lose the auction when a high bid of  $v$  would have had him earn positive surplus. Next, consider a deviation  $\beta'$  that has buyer  $i$  submit a high bid of  $v$ , but has him bid up to  $v$  in more than one bid. Such a deviation affect buyer  $i$ 's surplus only if (i)  $v \geq P_j(\Gamma) + d$ ;

and (ii) bidding in accordance with  $\beta'$  is perceived by aware buyers to be consistent with  $\alpha^*$ . In this case, should the high bidder in the paired auction  $j'$  be an aware buyer and  $Y_{j'} = P_j(\Gamma)$ , then upon reaching a state in which both auctions have a standing price of  $P_j(\Gamma)$ , the highest valuation aware buyer may, with positive probability bid first in auction  $j$ , raising the price to  $P_j(\Gamma) + d$  (in accordance with the third point 3 in Lemma 1). Of course, such a bid may have been placed with positive probability even if  $i$  had bid in accordance with  $\beta^*$ . It remains to show that there does not exist a strategy  $\beta'$  that calls  $i$  to bid up to  $v$  while minimizing the probability that  $i$ 's price is  $P_j(\Gamma) + d$ .

There are two cases to consider. In the first, suppose that  $v$  is at least  $2d$  more than the standing price in  $j$  at stage  $\Gamma$ . Suppose further that a bid of  $v$  makes  $i$  the high bidder in  $j$ . If not, what follows is irrelevant. If  $i$  bids  $v$  in  $\Gamma$ , then if the standing price increases by at least  $2d$ , buyers will infer that  $i$ 's bidding is inconsistent with  $\alpha^*$ . If  $i$ 's bid of  $v$  does not immediately raise the standing price by  $2d$ , then as the standing price in  $j$  is bid up to  $P_j(\Gamma)$  in subsequent rounds of bidding, once the standing price increases by  $2d$  without a change in high bidder, buyers will make the same inference as if the standing price had been raised by  $2d$  immediately following  $i$ 's bid. Thus, any  $\beta'$  such that  $i$ 's bidding according to  $\beta'$  causes aware buyers to infer that  $i$ 's bidding is inconsistent with  $\alpha^*$ , leaves them with the same beliefs regarding  $i$ 's high bid once  $P_j(\Gamma)$  is reached as if  $i$  bids according to  $\beta^*$ . In the second case,  $v$  is only  $d$  greater than the standing price in  $j$  at stage  $\Gamma$ . In this case,  $\beta^*$  is the only strategy available to  $i$  that has him bid  $v$  in auction  $j$ .

Next, consider buyer  $i$ 's strategy in stage  $\Gamma$  where he had already entered prior to stage  $\Gamma$ . If  $i$  had already bid  $v$ , then by arguments given previously, any additional bidding only serves to increase the probability that  $i$  wins  $j$  at a price above  $v$ . If  $i$  had previously bid above  $v$ , then any additional bids would have to be placed in auction  $j$  above his previous high bid. Such bidding can only serve to increase the probability that  $i$  wins in  $j$  at a price above  $v$ . If  $i$  had followed any other strategy such that his high

bid was below  $v$ , then if his previous maximum bid were to hold up, then bidding  $v$  in stage  $\Gamma$  would have no effect on the outcome. However, even if his previous maximum bid were a high bid, there is a probability that it won't hold up at a price strictly below  $v$ . In this case,  $i$  losses out on a profitable transaction. It remains to show that if  $i$ 's previous maximum were below  $v$ , then no alternative strategy  $\beta'$  that has  $i$  bid up to  $v$  in  $\Gamma$ , has  $i$  pay  $P_j(\Gamma)$  with a higher probability, as opposed to  $P_j(\Gamma) + d$ . The arguments are identical to those given for the case in which  $i$  had not entered prior to stage  $\Gamma$ . ■

## A.2 Proof of Corollary 1

In characterizing the difference in ending prices, we distinguish between those auctions in which the price is determined by solely by unaware buyers (i.e. the two highest bidders are unaware buyers) from those that are not. Define  $\Upsilon$  such that:

$$\Upsilon = \{j | P_j > Y_j^u\},$$

where  $Y_j^u$  denotes the second-highest valuation amongst all unaware buyer buyers in auction  $j$  if there are at least two unaware buyers in auction  $j$ , zero otherwise. Let  $\bar{\Upsilon}$  denote the complement of  $\Upsilon$ . If, under the hypothesis, we restrict attention to states at the conclusion of the bidding such that both auctions have standing prices of at least  $S_H + e$ , there are four states to consider:

- $\{L, H \in \Upsilon\}$ , in which case,  $P_L = P_H$ ;
- $\{L \in \Upsilon, H \in \bar{\Upsilon}\}$ , in which case,  $P_L < P_H$ ;
- $\{L \in \bar{\Upsilon}, H \in \Upsilon\}$ , in which case,  $P_H < P_L$ ;
- $\{L, H \in \bar{\Upsilon}\}$ , in which case, the comparison between  $P_L$  and  $P_H$  is ambiguous.

Let  $\Pi_{\Upsilon\bar{\Upsilon}}$  denote the probability of the event  $\{L \in \Upsilon, H \in \bar{\Upsilon}\}$ . The notation for the probability of the remaining events as enumerated above follow this construction. In calculating the probability of each state, we condition on there being a totla of  $m$  potential buyers in the market. Each of the  $m$  buyers, with probability  $\mu$  is uninformed, and with probability  $1 - \mu$ , is informed. Conditional upon being informed, with probability  $\rho$ , the buyer enters auction  $L$ , and with probability  $1 - \rho$ , enters auction  $H$ . Let  $N\{k_L, k_H; m, \mu, \rho\}$  denote the probability of there being  $k_L$  unaware buyers in auction  $L$  and  $k_H$  unaware buyers in auction  $H$ , given  $m$ ,  $\mu$ , and  $\rho$ . We will often express  $N$  in shorthand without the final three arguments removed. We have that:

$$N\{k_L, k_H\} = \binom{m}{k_L + k_H} \binom{k_L + k_H}{k_L} \mu^{k_L + k_H} (1 - \mu)^{m - k_L - k_H} \rho^{k_L} (1 - \rho)^{k_H}.$$

In calculating the expected prices across the various states, we make use of order statistics. Using standard notation for order statistics, let  $X_{(k:n)}$  denote  $k$ th highest value of  $n$  independent values. For our purposes,  $k$  and  $n$  denote the number of buyers participating in a given auction, with the ordering being taken over their valuations. Noting that the price difference is zero when  $L, H \in \Upsilon$ , we then have that:

$$\begin{aligned} D(S_H) &= E[P_L - P_H | P_L, P_H \geq S_H + e] \\ &= \sum_{k_H=0}^1 \sum_{k_L=2}^{m-k_H} E[X_{(2:k_L)} - X_{(2:m-k_L)} | L \in \Upsilon, H \in \bar{\Upsilon}, k_L, k_H] \Pi_{\Upsilon\bar{\Upsilon}} N\{k_L, k_H\} \\ &\quad + \sum_{k_L=0}^1 \sum_{k_H=2}^{m-k_L} E[X_{(2:m-k_H)} - X_{(2:k_H)} | L \in \bar{\Upsilon}, H \in \Upsilon, k_L, k_H] \Pi_{\bar{\Upsilon}\Upsilon} N\{k_L, k_H\} \\ &\quad + \sum_{k_H=2}^{m-2} \sum_{k_L=2}^{m-k_H} E[X_{(2:m-k_H)} - X_{(2:k_H)} | L, H \in \bar{\Upsilon}, k_L, k_H] \Pi_{\bar{\Upsilon}\bar{\Upsilon}} N\{k_L, k_H\}. \quad (5) \end{aligned}$$

Note that when  $N\{k_L, k_H\} = N\{k_H, k_L\}$ , the following are true:

1. The second term in the three-component summation of  $D(S_H)$  is the negative of the first; and
2. The third term is zero.

We have that  $N\{k_L, k_H\} = N\{k_H, k_L\}$  if  $\rho = 1/2$ .

### A.3 Proof of Corollary 2

By definition, an auction can be characterized as a divergent price auction if it can be inferred from the ending prices of the two auctions that the second-highest valuation of all buyers in the divergent auction exceeds that of the paired auction by at least  $d$ . Let the event  $j \in \Delta$  if auction  $j$  is a divergent auction. It follows that:

$$P\{j = L | j \in \Delta\} = \frac{P\{j \in \Delta | j = L\} P\{j = L\}}{P\{j \in \Delta | j = L\} P\{j = L\} + P\{j \in \Delta | j = H\} P\{j = H\}}.$$

Since  $P\{j = L\} = P\{j = H\} = 1/2$ , given the matched-pairs design, it follows that,

$$P\{j = L | j \in \Delta\} = P\{j = L | j \in \Delta\} = 1/2$$

if  $P\{j \in \Delta | j = L\} = P\{j \in \Delta | j = H\}$ . We have that for any  $m_L$  and  $m_H$  such that  $m_L + m_H = m$ ,

$$P\{j \in \Delta | j = L\} = P\{X_{(2:m_L)} \geq X_{(2:m_H)} + d | X_{(2:m_L)} \geq S_H + e\}. \quad (6)$$

Under the assumption that  $\rho = 1/2$ , we have that  $P\{k_L, k_H\} = P\{k_H, k_L\}$ . Therefore the conditional distribution of the second order statistic in auction  $L$ , conditional on being at least  $S_H + e$  is identical to that of auction  $H$ , which implies our result.

## A.4 Proof of Proposition 2

The model presented in Section 2.2 is identical to that of P&S, except for the inclusion of  $\pi \geq 0$ . We can think of P&S' model as a special case of our model in which  $\pi \geq 0$ . In that case, let  $\alpha^0$  denote the proposed equilibrium strategy under P&S as a special case of  $\alpha^\pi$ . It is evident that the comparison of buyers' payoffs across strategies is isomorphic to the inclusion of  $\pi$  once  $\pi$ -shifted starting prices are accounted for. Therefore, if  $\alpha^0$  constitutes a perfect-Bayesian equilibrium when  $\pi = 0$ , so too does  $\alpha^\pi$  when  $\pi > 0$ .

## A.5 Proof of Corollary 3

When  $\pi = 0$ , the equilibrium corresponding to  $\alpha^0$  is such that if  $P_L \geq S_H$ , then  $P_L = P_H$ . This is because once the starting price in auction  $L$  reaches  $S_H$ , the next bid will, with positive probability, be in auction  $H$ . If the next bid is placed in auction  $H$ , the buyer that placed the bid will pass as the bid would make her the high bidder in auction  $H$ . If not, then the buyer will become the high bidder in auction  $L$ , since she would only bid in auction  $L$  if the standing price had increased once since the last change in high bidder. In that case, the next buyer to bid will bid in auction  $H$ . In either case, the first bid in auction  $H$  will occur when the two auction have equal standing prices. From this point on, the two auctions' standing prices will continue to be equal until the bidding concludes, though their high bids may differ by at most  $d$ .

Now consider the equilibrium corresponding to  $\alpha^\pi$  when  $\pi > 0$ . Following  $\alpha^\pi$ , the first bid will be placed in auction  $H$  when the standing price in auction  $L$  is  $S_H + \pi$ . Following the first bid in auction  $H$ , the two auctions' standing prices will continue to differ by exactly  $\pi$ , though their high bids may differ by at most  $\pi + d$ . It follows that if  $P_H \geq S_H + e$ , then  $P_L = P_H + \pi$ .

## A.6 Proof of Corollary 4

Given the results of Corollary 3 if  $P_L > S_H + e$ , then  $P_L - P_H > e$  and auction  $L$  is a divergent auction. If  $P_L = S_H + e$ , then neither auction is a divergent auction. If,  $P_H \geq S_H + e$ , then  $P_L - P_H = \pi > e$ , so that again, auction  $L$  is a divergent auction.

## B Additional Figures

### Brand New!! Factory sealed.

This widescreen version of {insert name} is brand new in the original shrink wrap.

Payment by Pay Pal and money orders.

Ships anywhere in the U.S. and territories via first-class mail for a flat fee of \$3.00.

No combined shipping on multiple items. Sorry.

Will ship within 48 hours after receipt of payment.

Happy Bidding.

Figure 2.2: eBay listing, item description

Search term is "Cashflow 101," sorted by "Time: ending soonest," 66 results			Search term is "Cashflow," sorted by "Time: ending soonest," 245 results		
1	New CASHFLOW 101 BOARDGAME RICH DAD + READY STOCK	B	Rich Dad's Cashflow Quadrant by Robert T. Kiyosaki, ...	A	
2	Cashflow 101 investment game Rich Poor Dad Kiyosaki NEW	B	5 x MEI Cashflow Bill Acceptor SC6607R Frame 252007032	A	
3	CASHFLOW THE E-GAME (101 & 202)	A	New CASHFLOW 101 BOARDGAME RICH DAD + READY STOCK	B	
4	CASHFLOW THE E-GAME (101 & 202)	A	Cashflow 101 investment game Rich Poor Dad Kiyosaki NEW	B	
5	RICH DAD CASHFLOW 101 BOARDGAME w/BOOK & AUDIO GUIDE	A	Rich Dad's Cashflow Quadrant: Robert T. Kiyosaki, Shar	A,B	
6	NEW SEAL CASHFLOW 101 BOARD GAME RICH POOR DAD KIYOSAKI	B	Ron Legend's Quick Start Book Camp+ Cashflow Manuals\$\$	A,B	
7	NEW RICH POOR DAD CASHFLOW 101 BOARDGAME KIYOSAKI	A	CASHFLOW THE E-GAME (101 & 202)	A	
8	NEW CASHFLOW 202 + 101 BOARDGAME RICH POOR DAD KIYOSAKI	A	CASHFLOW THE E-GAME (101 & 202)	A	
9	New Seal Cashflow 101 Board Game Express 3 Day Ship +	B	Rich Dad's Cashflow Quadrant by Robert T. Kiyosaki, ...	A	
10	NEW RICH POOR DAD CASHFLOW 101 BOARDGAME KIYOSAKI	A	RICH DAD CASHFLOW 101 BOARDGAME w/BOOK & AUDIO GUIDE	A	

Figure 2.3: Cashflow search results, sorted by end time

Search term is "Cashflow 101," sorted by "Price + Shipping: Lowest First," 66 results			Search term is "Cashflow," sorted by "Price + Shipping: Lowest first," 245 results		
1	NEW RICH POOR DAD CASHFLOW 101 BOARDGAME KIYOSAKI	A	NEW RICH POOR DAD CASHFLOW 101 BOARDGAME KIYOSAKI	A	
2	RICH POOR DAD CASHFLOW 202 BOARDGAME KIYOSAKI FOR 101	A	RICH POOR DAD CASHFLOW 202 BOARDGAME KIYOSAKI FOR 101	A	
3	Cashflow 101 Robert Kiyosaki 3 cassettes	A	Cashflow Quadrant (Rich Dad's Advisers)	A	
4	Rich Dad's Cashflow 101	A	Squash, Cashflow, 25 seeds, Vegetable Seed	B	
5	NEW RICH POOR DAD CASHFLOW 101 BOARDGAME KIYOSAKI	A	The Cashflow Quadrant by Robert T. Kiyosaki, Sharon L.	A	
6	Cashflow 101	A	Rich Dad's Cashflow Quadrant by Robert T. Kiyosaki, ...	A	
7	Cashflow 101 +EXTRAS bonus Robert Kiyosaki Rich Dad NR	A	Rich Dad's Cashflow Quadrant Audiobook, Kiyosaki	A	
8	CASHFLOW 101 Rich Dad Board Game Real Estate Investor	A	R&B, CASHFLOW Love Education 12" Single Vinyl Record.	A	
9	CASHFLOW E-GAME 101 & 202	A	Rich Dad's Cashflow Quadrant by Robert T. Kiyosaki	A	
10	Cashflow 101 Board Game Complete Good Condition	A	Cashflow Quadrant: Rich Dad's Guide to Financial F	B	

Figure 2.4: Cashflow search results, sorted by lowest price



Search term is "Cashflow 101," sorted by "Price + Shipping: Highest First," 66 results			Search term is "Cashflow," sorted by "Price + Shipping: Highest first," 245 results		
1	NEW CASHFLOW 202 + 101 BOARDGAME RICH POOR DAD KIYOSAKI	B	CASHFLOW CASHFLOW original germany 1986 CD cameo	B	
2	Cashflow 101 Board Game Financial Rich Poor Dad Finance	B	CASHFLOW CASHFLOW Same s/t 1986 WEST GERMANY CD Cameo	B	
3	CASHFLOW 101 Board Game Rich Dad Poor Dad KIYOSAKI	B	MEI NCR CASHFLOW CURRENCY BILL ACCEPTOR 252017140 NEW!	B	
4	GENUINE CASHFLOW 101 Robert Kiyosaki Rich Poor Dad	B	CASHFLOW CASHFLOW (1986) @@ Original Germany CD Cameo	B	
5	RICH DAD POOR DAD CASHFLOW 101 GAME BOARD - NEW	B	NEW CASHFLOW 202 + 101 BOARDGAME RICH POOR DAD KIYOSAKI	B	
6	Cashflow 101 investment game Rich Poor Dad Kiyosaki NEW	B	MARS MEI Cashflow Series 7000 MDB COIN ACCEPTOR	B	
7	New Rich Dad Robert Kiyosaki CashFlow 101 Board Game	B	MEI NCR CASHFLOW MULTI-PROTOCOL CF9529e COIN ACCEPTOR	B	
8	Cashflow 101 USED ONCE Complete Rich Dad + CD's Sealed	B	Mars MEI Cashflow 7512i Coin Changer Refurbisher	B	
9	Rich Dad's Cashflow 101 AND Cashflow 202 Board Games!	A,B	Cashflow 101 Board Game Financial Rich Poor Dad Finance	B	
10	BRAND NEW SEAL CASHFLOW 101 BOARD GAME WITH 3 AudioCDs	B	CASHFLOW 101 Board Game Rich Dad Poor Dad KIYOSAKI	B	

Figure 2.5: Cashflow search results, sorted by highest price

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# CHAPTER 3

## INFORMATIONAL SHILL BIDDING

### 1 Introduction

The seminal work of Akerlof (1970) showed how informational asymmetry can erode the proper functioning of markets. Perhaps nowhere is informational asymmetry more apparent than in the sale of goods online where quality and authenticity are difficult to verify and the virtual identities of agents carry little weight. It should be no surprise that while the volume of transactions taking place online continue to grow, so do reports of abuse. Complaints of internet fraud continue to be the most prevalent source of fraud investigated by the Federal Trade Commission.<sup>1</sup>

A particularly injurious form of internet fraud involves a dishonest seller intentionally misrepresenting the condition or authenticity of an item so as to deceive potential buyers. Such deception is typically carried out through the illegal practice of shill bidding, whereby a seller in an online auction, often with the help of an accomplice, bids on his own item. In one high profile case from 2000, three men were indicted for inflating bids in attempt to sell a poor rendering of a supposed Richard Diebenkorn painting on eBay.<sup>2</sup> In the description of the item which accompanied the listing, the seller claimed ignorance of the painting's origins, saying he had found it at a garage sale and simply wondered whether it had any resale value. The description appeared very forthright, acknowledging the presence of minor scratches and tears, which were illustrated in a number of close-ups of the corners. One of the close-ups clearly revealed the name "Richard Diebenkorn" scrawled along the bottom-left corner. At no point did the seller

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<sup>1</sup>"Internet Auctions: A Guide for Buyers and Sellers." <http://www.ftc.gov>

<sup>2</sup>The story was chronicled by Judith Dobrzynski in a multi-part series in the New York Times, May 9 - 11, 2000.

mention the name of the late abstract expressionist Richard Diebenkorn.

Speculation that this garage-sale giveaway was in fact a lost work of the great modern master began to grow in online message boards. The steady rise in the auction's standing price, a result of the over 50 shill bids placed by the three codefendants in the case, served to further substantiate such speculation. The painting ultimately sold to a Dutch collector for \$135,805 before the scam was uncovered and the transaction voided, all without the seller ever claiming the painting to be a genuine Diebenkorn! Evidently, the escalation of the price through shill bidding was enough to convince this and other serious buyers that the item was genuine.

Though eBay continues to take steps to curb the practice, it is clear that such fraud persists, perpetuated with the aid of shill bidding. In March of 2008, the FBI uncovered an international ring of art forgers selling art on eBay as well as other venues.<sup>3</sup> In these cases, shill bidding was shown to be an integral component of carrying out the deception in online settings. In spite of consumer groups' warnings of the possibility of such fraud, buyers continue to bid on undocumented works of art, antiques, and collectibles, all *common value items*, with the hope of acquiring a treasured item undervalued by its current owner. This use of the platform as something of an online garage sale has proven critical to eBay's success since its inception. From the standpoint of eBay and other similar platforms, it is important to understand the economic factors that give rise to shill bidding in order to determine how best to preserve the integrity of the auction platform.

This paper models an auction environment within a game-theoretic context in order to shed light on the economic factors which allow shill bidding to persist. The model considers the sale of a single common value item online where market participants are asymmetrically informed as to the authenticity of the item. The authenticity of the item

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<sup>3</sup>"Seven charged in 'eBay art scam'," *BBC News*, March 20, 2008: Accessed from <http://news.bbc.co.uk/2/hi/americas/7306366.stm> September 15, 2008.

is modeled as a binary variable,  $Q \in \{Q^L, Q^H\}$ , which can be thought of as high quality versus low quality or genuine versus fake. To capture the possibility of the item being undervalued by its owner, suppose that the seller can either be *informed*, in which case he knows  $Q$  with certainty, or *uninformed*, in which case he assigns some probability to the event that the item is genuine. The demand side of the market consists of a single uninformed buyer, who shares the initial beliefs of an uninformed seller and whose valuation depends upon the common value element,  $Q$ , and an idiosyncratic taste parameter,  $x$ , which is private information.<sup>4</sup> In the event that the item is genuine, then with some probability, the market may also include an *informed buyer*, who knows  $Q$  with certainty and whose valuation depends only on  $Q$ . To facilitate the analogy between our stylized model and the real-world environment in which these auctions take place, in what follows, the two types of bidders are referred to as *collectors*, if uninformed, and *dealers*, if informed.

This setup is meant to capture the various informational asymmetries of the online auction environment in which a collector hopes to obtain a genuine article from a seller who does not know its true value. In the Diebenkorn case, the Dutch buyer seemingly responded to the bids of others and raised his bid accordingly. Such bidding can only be justified if he believes that other bidders know something about the authenticity of the item that he does not. The presence of dealers in our model serves this purpose.

The sales mechanism is modeled as a *dynamic second-price auction* in which bidding takes place over two discrete stages until a set end time.<sup>5</sup> The multi-stage feature

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<sup>4</sup>Engelbrecht-Wiggans, Milgrom, and Weber (1983) show that the indiosyncratic component is important to the setup. Without it, the uninformed buyer cannot earn positive surplus and as such has no incentive to participate unless his valuation exceeds that of the informed buyer.

<sup>5</sup>Dynamic second-price auction is the name commonly given to the ascending-price auction employed by eBay. In a dynamic second-price auction, buyers are free to bid as many times as they choose during a predetermined window set by the seller. At any point during the auction, buyers can observe the identity of the high bidder and the standing price, which is equal to: (i) the starting price if one or fewer bids have been received; or (ii) the second-highest bid plus a small increment if two or more bids have been received. Buyers also observe the number of bids and the number of distinct bidders at any point during the auction. The standing price is updated in real time until the time runs out. The winner is the high bidder when the time runs out and pays a price equal to the standing price at the auction's end.

allows for bids placed in stage 1 to provide information to collectors which they can incorporate into their bidding strategy in stage 2.<sup>6</sup> In particular, a collector who believes that a stage-1 bid was placed by a dealer must then also believe the item to be genuine, and thus be willing to pay more than what he would have were the authenticity unknown. Therefore, a shill bid placed by a fraudulent seller (an informed seller of a fake) for the purpose of deceiving collectors into thinking that the bid was placed by a dealer, is referred to as an *informational shill bid*. I establish an equilibrium of the game characterized by the following set of behaviors:

- If present in the market, a dealer bids in stage 1 with positive probability;
- If present in the market, a fraudulent seller disguises himself as a dealer by bidding in stage 1;
- Upon observing a bid by another participant in stage 1, a collector increases his assessment of the item;
- A subset of collectors (distinguished by their idiosyncratic taste parameter) increase their bid in stage 2 if and only if they observe a bid in stage 1.

In the aforementioned *informational shill bidding equilibrium* (“ISBE”), a collector may, with positive probability, obtain a fake at a price above what he would have been willing to pay had he not observed a bid in stage 1. This is likely the position in which the Dutch buyer in the Diebenkorn case found himself. Thus, the model establishes that such an outcome can occur in equilibrium even when potential buyers understand the potential for such fraud. For this to occur, however, it must be the case that fraudulent sellers make up a sufficiently small proportion of all sellers. Otherwise, the observation of a bid by an opposing bidder may actually negatively influence a buyer’s assessment as to the genuineness of the item.

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<sup>6</sup>Any more than two stages of bidding would be superfluous under this setup.



An additional restriction in bringing about an ISBE is that the starting price chosen by an uninformed seller, which is then mimicked by fraudulent sellers posing as uninformed sellers, must be sufficiently large that a set of collectors abstain from bidding in stage 1, thereby only bidding in stage 2 upon observing a bid in stage 1. Given this result, it is necessary to understand all possible subgames following from all feasible starting prices in order to characterize the equilibrium starting price. First, I find that there exists such a subgame equilibrium in which a collector's assessment of the item upon observing a bid in stage 1, is not high enough to justify increasing his bid in stage 2. In this case, all collectors bid only in stage 1 an amount that takes into account the ex-ante expected value of the item. Fraudulent sellers may still shill under these circumstances, but only for the purpose of extracting greater surplus, given the bidding of collectors, and not for the purpose of influencing the bidding of collectors. Thus, this alternative equilibrium is referred to as a *non-informational shill bidding equilibrium* ("Non-ISBE"). Next, a subgame equilibrium is established in which an uninformed seller sets a starting price in excess of a dealer's valuation of a genuine item. This *no shill bidding equilibrium* ("NSBE") equilibrium precludes bidding by dealers and by extension, shill bidding by fraudulent sellers.

A numerical comparison highlights the parameterizations that rationalize the three types of equilibria, respectively. The parameterizations giving rise to the ISBE and Non-ISBE are shown to be nearly complementary to one another, except for a slight overlap in which either type of equilibrium is possible. As the proportion of fraudulent sellers increases, the set of all other parameters rationalizing the ISBE shrinks while that of the Non-ISBE grows. The parameterizations giving rise to a NSBE is a subset of that giving rise to an ISBE. Where the parameterizations giving rise to the ISBE and the Non-ISBE overlap, the Non-ISBE gives rise to greater surplus than the ISBE within the range of parameters considered. Thus, while some collector types benefit from the information gleaned from stage 1 bids within the ISBE, the overall effect of informational shill bid-

ding is negative as it causes other collector types to simply not participate, which is partially explained by a higher starting price within the ISBE. Further, the fact that the Non-ISBE outperforms the ISBE in total surplus gives rise to a very counter-intuitive result that an increase in the proportion of fraudulent sellers, though negatively affecting total surplus in either equilibrium type, can actually result in an increase in total surplus in the event it causes players to switch from playing strategies consistent with the ISBE to that of the Non-ISBE.

Of the existing literature on shill bidding, the study most closely resembling this one is Chakraborty and Kosmopoulou (2004), which considers the sale of a pure common value item to buyers with symmetric, but imperfect, information. In Chakraborty and Kosmopoulou (2004), buyers update their beliefs over the authenticity of the item in response to the number of competing bidders, which in the equilibrium they consider, is positively correlated with the likelihood that the item is genuine. Similar to the result shown herein, shill bids are only informative when the proportion of would-be shillers is sufficiently small. I, nevertheless, argue that the current study offers two advantages over Chakraborty and Kosmopoulou (2004). First, Chakraborty and Kosmopoulou (2004) model the sales mechanism as a *button-push auction* of Milgrom and Weber (1982), in which the price ascends automatically until all but the last bidder have released their button and dropped out. This modeling assumption abstracts away from the decision of whether or not to bid in the early stages of the auction if bidding provides valuable information to one's opponents. In the button-push auction, a buyer is required to keep his button depressed if he is to be allowed to participate in later stages. The assumption also abstracts away from the amount a buyer would choose to bid as the standing price is increased automatically until only one bidder remains. This assumption restricts the strategy set of buyers in a way that prevents additional information from being conveyed. By giving buyers a choice as to whether or not to bid in each stage, the auction mechanism considered herein more closely resembles the dynamic second-

price auction employed by eBay. Second, Chakraborty and Kosmopoulou (2004) model all buyers as having identical valuations and identical beliefs. As such, a buyer's expected surplus is zero whether or not shill bidding is informative, thus precluding any meaningful comparison of consumer surplus.

Much of the remaining literature on shill bidding restricts attention to the sale of private value goods.<sup>7</sup> In these models, shill bidding serves to extract additional surplus from buyers in ascending-price auctions by exploiting the difference between the standing price and the high bidder's valuation, a practice dubbed *competitive shilling* by Kauffman and Wood (2005). Kauffman and Wood (2005) distinguish between competitive shilling and *reserve price shilling*, whereby a seller uses a single shill bid to serve the purpose of a reserve price while shielding themselves from eBay's listing fees which discourage the use of reserve prices. While not mutually exclusive, competitive shilling and reserve price shilling are means for a seller to increase his or her surplus illegally; but when used in conjunction with the sale of a private value good, such behavior need not be inefficient. Izmalkov (2004) shows that shill bidding can be used to implement the optimal auction mechanism of Myerson (1981) when buyers' valuations are independently drawn from different distributions. In a pure private values setting, a seller restricted to using a single starting price and no shill bid may set the starting price too high from an efficiency standpoint. But when shill bidding can be used in conjunction with a starting price, the chosen starting price may be set lower in order to bring about an outcome that more closely resembles that of first-degree price discrimination. In contrast, informational shill bidding can only lead to inefficient outcomes and is thus a greater threat to the proper functioning of markets than are competitive shilling and reserve price shilling. The study of informational shill bidding is important in understanding the economic mechanisms at play in the fraudulent sale of the Diebenkorn

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<sup>7</sup>See Sinha and Greenleaf (2000), Wang, Hidvegi, and Whinston (2001), Izmalkov (2004), and Wang, Hidvegi, and Whinston (2004) for examples of models of shill bidding in which buyers know their valuations with certainty.

painting considered earlier as models of competitive and reserve price shilling cannot capture such behavior.

The paper is organized as follows: Section 2 presents the basic model and formalizes the concept of an informational shill bidding equilibrium; Section 3 demonstrates conditions under which an informational shill bidding equilibrium exists; Section 4 establishes two alternative equilibria among the set of all possible equilibria of the game and characterizes the necessary conditions of all equilibria; Section 5 illustrates the parameterizations giving rise to the various classes of equilibria within a numerical example; and Section 6 concludes.

## **2 Common Value Auction Model with Asymmetric Information**

The following describes the basic model used throughout the remainder of the paper and defines the concept of an ISBE.

Consider a single seller attempting to auction a single item to a set of potential buyers. The seller may be either “informed” or “uninformed” as to the authenticity of the item, which is represented by some quality parameter  $Q$ . An informed seller knows  $Q$  with certainty while an uninformed seller has only imperfect information over  $Q$ . Let  $\phi$  denote the proportion of sellers who are informed.

The identities of potential buyers are unknown prior to the start of the auction. As such, information is revealed only by the bidding. Each buyer has a private valuation over the item which depends, at least partially, on the realization of  $Q$ . A buyer may be either uninformed, a “collector,” or informed, a “dealer.” A dealer is assumed to know  $Q$  with certainty while a collector has only imperfect information over  $Q$ .

Consistent with the interpretation that the item be either genuine or a fake, the quality

parameter takes on one of two values,  $Q^H$  and  $Q^L$ . Assume that uninformed sellers and collectors hold common initial beliefs over the value of  $Q$ , denoted by  $\mu_0$ . Beliefs are updated over the course of the game, using Bayes' rule. Beliefs of collectors depend upon the information set which the player finds himself at, denoted  $I$ . Let the beliefs of a player with imperfect information and at information set  $I$ , be given by the function  $\mu$  such that:

$$\mu \{I\} = P \{Q = Q^H | I\}.$$

Given the interpretation of the informed buyers as dealers, who care only about the resale value of the item, assume that dealers' valuations depend only on  $Q$ . In this way, let a dealer's valuation be equal to  $Q$  and normalize the values of  $Q$  such that  $Q^L = 0$  and  $Q^H = 1$ .

Collectors' valuations are assumed to vary based upon personal taste. In this way, let  $x \in X$  denote some taste parameter, such that the valuation for buyer  $i$  with taste parameter  $x_i$ , in information set  $I$ , is denoted:

$$v(x_i; \mu \{I\}) = (1 + x_i) \mu \{I\} Q^H.$$

It is assumed that the  $x_i$  are independent and identically distributed over  $[0, 1]$ .

The valuation of an uninformed seller depends only on his information. Let  $\omega(\mu \{I\})$  denote the valuation of an uninformed seller who believes the item to be of quality  $Q^H$  with probability  $\mu$ , where  $\omega \in [0, 1]$  is common knowledge. Informed sellers of fake items, which are referred to as *fraudulent sellers*, have no value for the item. This assumption is consistent with the notion that if the fraudulent seller were to attempt to sell the item in a traditional market, it would be evident that the item was not genuine and as such would not sell for any price above  $Q^L$ , which has been normalized to zero.

To simplify the analysis, the number of bidders in the auction are limited to at most two: one collector and possibly one dealer. Regardless of the realization of  $Q$ , there will be a single collector in the market. If  $Q = Q^H$ , then with probability  $\gamma$ , there will be a dealer in the market; if  $Q = Q^L$ , dealers will not enter the market so the collector will be the only potential buyer. Neither the collector nor the seller know whether a dealer is present except by inferring from his bidding behavior. If present, the dealer knows that a single collector is present but does not know the collector's taste parameter,  $x$ , since it is private information.

This setup is meant to capture a situation in which buyers (both collectors and dealers) scour the listings in hopes of finding an item that is undervalued by its seller i.e. an uninformed seller. During the course of any auction, collectors can gain information as to the authenticity of the item upon observing the bidding of others i.e. dealers, as such bidders are thought to have superior information. Learning from the bidding of others in this way can lead to a collector raising his assessment of the item's value. But when it is known that collectors use information in this way, an incentive exists for fraudulent sellers to bid as a dealer would in order to deceive the collector.

The sales mechanism is a two-stage dynamic second-price auction. The timing of the game is as follows:

1. The seller lists the item for sale and posts a public starting price,  $S$ .
2. Stage 1 of the bidding: All buyers, including a shilling seller, may place a single bid that must exceed the starting price; all bids are recorded; the identity of the high bidder is revealed as are the number of bids, and the standing price, which is equal to the second highest bid if two or more bids have been placed, or the starting price if one or fewer bids have been placed.
3. Collectors and uninformed sellers update their beliefs based upon the information revealed following stage 1 of the bidding.

4. Stage 2 of the bidding: All buyers, including those who did not bid in stage 1, may place a single bid. Each bid may fail to be transmitted with probability  $\alpha \in (0, 1)$ . The identity of the winning bidder and the final price are revealed and the transaction is completed.

Table 3.1: Summary of notation

$Q$	the quality of the item, taking on values in $\{Q^L, Q^H\} = \{0, 1\}$
$\phi$	the proportion of informed sellers in the market
$\mu\{I\}$	a collector's beliefs as to the probability that $Q = Q^H$ at information set $I$ ; $\mu_0$ denotes his initial beliefs
$x$	a collector's taste parameter, which is uniformly distributed over $[0, 1]$
$v(x, \mu\{I\})$	a collector's valuation of the item, given taste parameter $x$ and beliefs $\mu\{I\}$ ; equal to $(1 + x)\mu\{I\}$
$\omega(\mu\{I\})$	an uninformed seller's valuation of the item, given beliefs $\mu\{I\}$
$\gamma$	the probability of there being a dealer in the market, conditional on $Q = Q^H$
$\alpha$	the probability that a bid placed in stage 2 of the auction fails to transmit

The two-stage bidding setup allows for early bids placed in stage 1 to convey information which can then be incorporated into the stage 2 bids of collectors. The probability of transmission error in stage 2 reflects the reality that to be sure that there is no time left in the auction for opposing bidders to bid in response to one's bid comes with the risk that one's own bid is not placed in time. Without this assumption, there would be no incentive for a dealer to bid in stage 1 as he would simply wait for stage

2 where his information would remain private until after the auction had ended.<sup>8</sup> Table 3.1 summarizes the notation that has been introduced in this section.

In considering the behavior of informed sellers of genuine items, it is assumed that these seller types can *distinguish* themselves from other seller types. Whether this takes the form of publishing the results of an appraisal or by signaling the item's quality through some other mechanism is not important as long as the information is conveyed to uninformed buyers as of the start of stage 1. The rationale for this assumption is justified in Section 3.

The concept of an *informational skill bidding equilibrium* (ISBE) can now be defined as follows:

**Definition 1** *An informational skill bidding equilibrium is an equilibrium of the two-stage dynamic second-price auction game characterized by the following:*

1. *Informed sellers of authentic articles distinguish themselves from all other seller types;*
2. *Fraudulent sellers choose a starting price so as to be indistinguishable from uninformed sellers to collectors;*
3. *Dealers, if present in the market, bid in stage 1 with positive probability;*
4. *Fraudulent sellers mimic the actions of dealers in stage 1;*
5. *Collectors update their beliefs in response to stage 1 bidding such that a higher standing price following stage 1 leads to a greater likelihood that a higher bid will be placed in stage 2.*

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<sup>8</sup>Roth and Ockenfels (2002) introduce this assumption as a way of modeling the end of auction rule in eBay.



### 3 Characterization of an ISBE

This section, begins by characterizing an ISBE. Such an equilibrium requires that dealers bid in stage 1 and that fraudulent sellers mimic that bid. In establishing an ISBE, the formal equilibrium concept used is *perfect Bayesian equilibrium*, which specifies the actions of each player at each possible information set as well as a set of beliefs over the types of all other players.

The analysis begins by considering the beliefs of a collector upon reaching information set  $I_1$ , the start of the auction. Upon entering an auction in which the item's true quality is not revealed, he infers that he is not in an auction for a genuine item being sold by an informed seller, an occurrence with probability  $\phi\mu_0$ . The possible realizations, conditional upon having reached  $I_1$ , are the following:

- Seller is uninformed and  $Q = Q^H$ : probability of  $(1 - \phi) \mu_0 / (1 - \phi\mu_0)$ ;
- Seller is uninformed and  $Q = Q^L$ : probability of  $(1 - \phi) (1 - \mu_0) / (1 - \phi\mu_0)$ ;
- and
- Seller is informed and  $Q = Q^L$ : probability of  $\phi (1 - \mu_0) / (1 - \phi\mu_0)$ .

Therefore, upon reaching the start of the auction, the buyer believes that the probability that the item is genuine is,

$$\mu \{I_1\} = \frac{(1 - \phi) \mu_0}{1 - \phi\mu_0}, \quad (1)$$

which is less than his initial belief of  $\mu_0$  for any  $\phi > 0$ .

Suppose that dealers are thought to bid some amount  $\beta$  in stage 1. Following stage 1, a collector's beliefs will be updated in a manner depending upon whether or not he observes a bid in stage 1 that is *indistinguishable* from a bid of  $\beta$ . Such an outcome occurs under the following circumstances:

- The collector does not bid in stage 1 and observes that a bid has been placed by another player. In this circumstance, the standing price would be  $S$ , regardless of the magnitude of the bid. Since the high bid is always unobservable to other players, the high bid could be any amount greater than  $S$ .
- The collector bids some amount less than  $\beta$  in stage 1 and finds that he is not the high bidder following stage 1. The standing price would be equal to the collector's bid. As in the previous case, the high bid would be unobservable, so it could be any amount greater than the collector's bid.
- The collector bids some amount greater than or equal to  $\beta$  in stage 1 and finds he is the high bidder following stage 1 and the standing price is  $\beta$ . As the buyer in question is the high bidder, the bid of the opposing bidder is observable.

In each of the aforementioned cases, the bidding of the opposing bidder is consistent with the equilibrium strategy that would be played by a dealer or mimicked by a fraudulent seller engaged in shill bidding. Let  $I^0$  denote this information set. The collector, upon finding himself in information set  $I^0$ , updates his beliefs as follows:

$$\mu \{I^0\} = \frac{\gamma \mu \{I_1\}}{\gamma \mu \{I_1\} + \phi (1 - \mu \{I_1\})}. \quad (2)$$

The derivation of equation (2) involves a direct application of Bayes' theorem:  $\gamma$  represents the probability of  $I^0$  being reached, conditional on  $Q = Q^H$  and conditional on  $I_1$ ;  $\phi$  represents the probability of  $I^0$  being reached, conditional on  $Q = Q^L$  and conditional on  $I_1$ .

A second information set that can be reached in equilibrium is one in which no bids are placed by opposing players. In equilibrium, this occurs when either the item is a fake but the seller is uninformed, or if the item is genuine, the seller is uninformed and no dealers are present. Let  $I'$  denote this information set. Upon finding himself in

information set  $I'$ , the collector updates his beliefs as follows:

$$\mu\{I'\} = \frac{(1 - \gamma) \mu\{I_1\}}{(1 - \gamma) \mu\{I_1\} + (1 - \phi)(1 - \mu\{I_1\})}, \quad (3)$$

where the construction is symmetric to that of equation (2). For the beliefs specified by equations (2) and (3) to be consistent with an ISBE, it must be the case that observing a bid consistent with the bidding of a dealer raises a collector's assessment of the item. It is straightforward to show that  $\mu\{I'\} < \mu\{I_1\} < \mu\{I^0\}$  if and only if  $\phi < \gamma$ .

The following lemma applies a standard result in auction theory to the current setting, which guides the analysis of equilibrium bidding strategies.

**Lemma 1** *All buyer types bid their valuations in stage 2 if they haven't already bid an amount greater than or equal to their stage-2 valuation in stage 1. Specifically,*

1. *A collector, upon finding himself in some information set  $I$  following stage 1, bids  $v(x; I)$  in stage 2 if  $v(x; I)$  is both greater than the standing price and his bid in stage 1.*
2. *A dealer bids  $Q^H$  in stage 2 if  $Q^H$  is greater than both the standing price and his stage 1 bid of  $\beta$ .*

The intuition for this result follows from the fact that bids in stage 2 have no strategic ability to alter the bidding of opposing bidders. As such, each potential buyer bids in a manner consistent with a pure private value setting.

The next result establishes that certain collector types, distinguished by their taste parameter, will bid in stage 2 in response to the information set reached following stage 1. In keeping with the intuition of the ISBE, it should be the case that those collectors that do respond to information revelation only respond to positive information. This is specified in the following condition. Before establishing the result, define

the following collector types:  $\bar{x}(Q^H; I^0) = \min \{x \mid v(x; I^0) = Q^H\}$  and  $\bar{x}(S; I') = \min \{x \mid v(x; I') = S\}$ .

**Condition 1** *In any ISBE, it is required that  $\bar{x}(Q^H; I^0) < \bar{x}(S; I')$ .*

Condition 1 requires that the lowest type that would find it profitable to bid should  $I'$  be reached within an ISBE,  $\bar{x}(S; I')$ , exceed the lowest type that would find it profitable to bid should  $I^0$  be reached,  $\bar{x}(Q^H; I^0)$ .<sup>9</sup> The logic for this follows from the definition of an ISBE which requires that a bid placed in stage 1 makes additional bids in stage 2 more likely. In an ISBE, upon observing a bid in stage 1, a collector believes that the item is more likely to be genuine than if he had not. At the same time, the observed bid raises the high bid from  $S$  to  $\beta$  and ultimately to  $Q^H$  should the dealer or fraudulent seller's stage 2 bid be transmitted. Therefore, Condition 1 requires that the observation of the bid in stage 1 has a large enough effect on collectors' beliefs such that the collector types whose valuations fall between  $\bar{x}(Q^H; I^0)$  and  $\bar{x}(S; I')$  find it profitable to bid in stage 2 following observing a bid in stage 1, but would not if they did not observe a bid.

The bidding of collectors under Condition 1 is characterized in what follows.

**Lemma 2** *Given  $\beta \in [S, Q^H]$ , there exists a cutoff  $x_\beta^*$  such that the collector's best response,  $b_\beta^0$ , is characterized as follows:*

- *If  $x \geq x_\beta^*$ , then bid  $v(x; \mu\{I^0\})$  in stage 1;*
- *If  $x < x_\beta^*$ , then, do not bid in stage 1; bid  $v(x; \mu\{I^0\})$  in stage 2 if and only if  $I^0$  is reached and if  $x \geq \bar{x}(\beta; I^0)$ ;*
- *Otherwise, do not bid in either stage.*

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<sup>9</sup>This assumes, for ease of discussion, that the dealer or fraudulent seller's bid of  $Q^H$  in stage 2 is transmitted. The proof of the following lemma takes into account that stage 2 bids face the risk of transmission failure.

The value  $x_\beta^*$  serves to distinguish collector types whose expected valuation of the item is sufficiently high as to make bidding in stage 1 profitable from those that only bid upon observing positive information in stage 1. When Condition 1 holds,  $x_\beta^*$  falls between  $\bar{x}(\beta; I^0)$  and  $\bar{x}(S; I')$ . Thus, the indifferent type,  $x_\beta^*$ , earns negative surplus should  $I'$  be reached. For buyers whose taste parameter falls between  $\bar{x}(Q^H; I^0)$  and  $x_\beta^*$ , it is only profitable to bid upon observing a bid in stage 1, which makes the possibility that the item is genuine more likely. It is these bidders for which shill bidding is intended to deceive.

Taking as given the behavior of collectors, the following result describes the bidding of dealers in equilibrium.

**Lemma 3** *When collectors bid according to  $b_\beta^0$ , a dealer's best response is characterized as follows: there exists an  $\alpha^* > 0$  such that for any  $\alpha \geq \alpha^*$ , a dealer bids  $Q^H$  in stage 1 with probability 1.*

The proof of Lemma 3 proceeds in two parts: first, showing that conditional on bidding in stage 1, the optimal bid is  $Q^H$ ; and second, that for  $\alpha$  sufficiently large, it is a best response to bid in stage 1 as opposed to stage 2. The first part follows from the fact that when collectors bid according to  $b_\beta^0$ , the magnitude of  $\beta$  has no effect on the collector's bid. If  $x \geq x_\beta^*$ , the collector does not bid in stage 2 and so his bidding is unaffected by the value of  $\beta$  (even if his assessment of the item is). If  $x < x_\beta^*$ , the collector believes himself to be in information set  $I^0$  regardless of the magnitude of the dealer's bid since he does not bid in stage 1 and cannot observe the magnitude of the high bid. Given no informational advantage to disguising his valuation in stage 1, bidding anything less than his valuation only serves to expose him to the risk of transmission failure upon attempting to increment his bid in stage 2. Given this result, let  $x^* \equiv x_{Q^H}^*$  and  $b^0 \equiv b_{Q^H}^0$ .

In deciding whether to bid  $Q^H$  in stage 1 or in stage 2, the dealer trades off a higher

probability of obtaining the item from bidding in stage 2, conditional on having his bid transmitted, against a higher probability of having his bid be transmitted in stage 1. As the probability of transmission failure gets larger, the latter consideration swamps the former. The value  $\alpha^*$  is the largest such value of  $\alpha$  that make a dealer indifferent between the two. The proof of the lemma further demonstrates that  $\alpha^* \leq .5$ .

Formally, a dealer bids in stage 1 with probability one if the following holds:

$$\bar{x}(Q^H; I^0) + \alpha [x^* - \bar{x}(Q^H; I^0)] \geq (1 - \alpha) x^*. \quad (4)$$

Inequality (4) expresses a dealer's trade-off in deciding whether to bid  $Q^H$  in stage 1 or stage 2. The dealer's surplus upon winning,  $1 - S$ , cancels out of both sides as  $S$  is necessarily the price he pays upon his obtaining the item. Thus the trade-off is over the probability of winning under the two possible actions. Bidding in stage 1, the dealer obtains the item if either the collector's preference parameter is not high enough to bid in stage 2 should  $I^0$  be reached, or if the collector's valuation is high enough to only bid if  $I^0$  is reached, but his bid fails to transmit. Upon bidding in stage 2, the dealer obtains the item if his own bid transmits and if the collector's preference parameter is not high enough to justify bidding in stage 1.

If inequality (4) does not hold, this does not imply the existence of an equilibrium in which dealers bid only in stage 2. To see this, assume that  $\alpha < .5$  and consider an equilibrium in which the inequality (4) is reversed. In such an equilibrium, dealers would find it unprofitable to bid in stage 1. Therefore, upon observing a bid in stage 1, a collector would infer that the bid had been placed by a fraudulent seller; this implies that  $\bar{x}(Q^H; I^0) = 1$ , thus violating Condition 1. It follows that an ISBE must have dealers bidding in stage 1 with positive probability. However, when the inequality in equation (4) is reversed, it is unprofitable for a dealer to bid in stage 1 when collectors believe that he will do so. Thus, an equilibrium in this case must be one in which collectors

believe dealers will bid in stage 1 with probability  $\delta < 1$  – a decrease in  $\delta$  increases a dealer's surplus from bidding in stage 1 – and that under those beliefs a dealer is just indifferent between bidding in stage 1 and bidding in stage 2. Indifference requires that equation (4) holds with equality, where collectors' beliefs in equations (2) and (3) must be revised accordingly to reflect the fact that  $\delta < 1$ .

A collector's beliefs, when dealers bid in stage 1 with probability  $\delta$  (while fraudulent sellers still bid in stage 1 with probability one), can be expressed as:<sup>10</sup>

$$\mu \{I^0; \delta\} = \frac{\gamma \delta \mu \{I_1\}}{\gamma \delta \mu \{I_1\} + \phi (1 - \mu \{I_1\})} \quad (5)$$

and

$$\mu \{I'; \delta\} = \frac{(1 - \gamma \delta) \mu \{I_1\}}{(1 - \gamma \delta) \mu \{I_1\} + (1 - \phi) (1 - \mu \{I_1\})}. \quad (6)$$

It is then straightforward to extend the derivation of  $b^0$ , the collector's bidding strategy, to the case where  $\delta < 1$  by expressing the collector's beliefs accordingly. Formally, let  $P \{I^0; \delta\}$  and  $P \{I'; \delta\}$  denote the probability that  $I^0$  and  $I'$ , respectively, are reached, given  $\delta$ . Further, let  $\Delta$  denote the probability of a dealer transmitting a bid in stage 2, conditional upon  $I'$  being reached.<sup>11</sup> Lastly, let  $\bar{x} (Q^H; I^0; \delta)$  and  $\bar{x} (S; I'; \delta)$  be as before, only now reflecting the fact that  $\delta$  may be less than unity. It follows that the cutoff collector type,  $x^* (\delta)$ , takes the form:

$$x^* (\delta) = \begin{cases} 0 & \text{if } \bar{x} (S; I'; \delta) = 0 \\ 1 & \text{if } \bar{x} (Q^H; I^0; \delta) = 1 \\ c + \xi S & \text{otherwise} \end{cases} \quad (7)$$

<sup>10</sup>It will be shown in what follows that as long as Condition 1 holds, fraudulent sellers will shill with probability one.

<sup>11</sup>This probability is zero if  $\delta = 1$ . Clearly  $\Delta$  is a function of  $\alpha$ ,  $\delta$  and  $\gamma \mu \{I_1\}$ , the probability of there being a dealer in the market.

where

$$c = \frac{\alpha P\{I^0; \delta\} + P\{I'; \delta\} \Delta}{[1 - \delta\gamma(1 - \alpha)] \mu\{I_1\}} - 1$$

and

$$\xi = \frac{P\{I'; \delta\} (1 - \Delta)}{[1 - \delta\gamma(1 - \alpha)] \mu\{I_1\}}.$$

Having established the bidding of potential buyers, consider the strategy of a fraudulent seller. It is assumed that a fraudulent seller cannot take any action that would have him be identified as a fraudulent seller. This restriction is justified on the grounds that were he to be identified as a fraudulent seller, the auction would be voided and terminated immediately.<sup>12</sup> This restriction affects a fraudulent seller's choice of starting price as well as his shill bidding decision. Since it has been assumed that informed sellers of genuine items have already distinguished themselves as such – either by directly verifying the authenticity of the item or through some other mechanism – the fraudulent seller is left with no other choice of starting price other than  $S$ , the uninformed seller's starting price.

In deciding whether or not to shill bid, he chooses between a bid of  $Q^H$ , a strategy of not bidding at all and a strategy of bidding in only stage 2 if  $\delta < 1$  (since a bid in stage 2 would have him potentially revealed as a shiller if dealers only bid in stage 1). A shill bid of  $Q^H$  in stage 1 puts the collector in information set  $I^0$ . Thus, the fraudulent seller gains a sale at a price of  $Q^H$  with probability  $1 - \alpha$  when the collector's taste parameter is between  $\bar{x}(Q^H; I^0; \delta)$  and  $x^*(\delta)$ , whereas he otherwise would have received no bid. When the collector's taste parameter is greater than or equal to  $x^*$ , the collector bids in stage 1 regardless of the fraudulent seller's shilling decision, however placing a shill bid of  $Q^H$  raises the price paid by the collector from  $S$  to  $Q^H$ . Lastly, when the collector's taste parameter is below  $\bar{x}(Q^H; I^0; \delta)$ , the auction would not have resulted in a sale

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<sup>12</sup>This assumption is equivalent to assuming that the cost of being identified as a fraudulent seller is greater than  $2Q^H$ , the maximal gain from carrying out the fraud.



regardless of the fraudulent seller's shilling decision. As the fraudulent seller cannot be made worse off, there is no incentive for a fraudulent seller to deviate from the proposed shilling strategy.

It remains to characterize the uninformed seller's choice of starting price,  $S^0$ . If Condition 1 is satisfied at  $S^0$  and  $\delta^*$ , then the subgame following the seller's announcement of  $S^0$  is consistent with an ISBE. Notice that if  $S^0$  were to equal  $Q^H$ , Condition 1 would necessarily be satisfied. Thus, Condition 1 requires that  $S^0$  is not too much lower than  $Q^H$ . This is accomplished when the seller's valuation is sufficiently high.

The following proposition characterizes an ISBE in the subgame in which the uninformed seller's starting price,  $S^0$  satisfies the aforementioned condition. It must be noted that this does not imply that such a starting price is globally optimal over all possible starting prices that the uninformed seller can choose. In order to solve for the global optimum, it remains to characterize the subgame equilibria that are consistent with each possible choice of  $S$ . This task is taken up in the following section.

In what follows, let  $\omega'$  denote uninformed seller's expected valuation, conditional on the item going unsold in an auction in which other players play according to the strategies specified in the equilibrium. It can be shown that  $S^0$  is increasing in  $\omega'$ , which implies the last result.

**Proposition 1 ISBE.** *If the seller's optimal starting price gives rise to the following, then the following constitute an equilibrium:*

- *Dealers bid  $Q^H$  in stage 1 with probability 1 if the following holds:*

$$\bar{x}(Q^H; I^0; 1) + \alpha [x^*(1) - \bar{x}(Q^H; I^0; 1)] \geq (1 - \alpha) x^*(1);$$

*otherwise, they randomize, bidding  $Q^H$  in stage 1 with probability  $\delta^*$  and bidding*

$Q^H$  in stage 2 with probability  $1 - \delta^*$ , where  $\delta^*$  satisfies:

$$\bar{x}(Q^H; I^0; \delta^*) + \alpha [x^*(\delta^*) - \bar{x}(Q^H; I^0; \delta^*)] = (1 - \alpha) x^*(\delta^*).$$

- Collectors update their beliefs following stage 1 according to equations (5) and (6) and bid according to  $b^0$ .
- Fraudulent sellers bid  $Q^H$  in stage 1 with probability 1; and
- Both uninformed sellers and fraudulent sellers set a starting price of  $S^0$  solving:

$$\begin{aligned} \max_{S \in [0,1]} & \gamma \mu_0 ([1 - x^*(\delta^*)] + (1 - \alpha) [x^*(\delta^*) - \bar{x}(Q^H; I^0; \delta^*)]) \quad (8) \\ & + \gamma \mu_0 S (\alpha [x^*(\delta^*) - \bar{x}(Q^H; I^0; \delta^*)] + \bar{x}(Q^H; I^0; \delta^*)) \\ & + (1 - \gamma \mu_0) [S (1 - x^*(\delta^*)) + \omega' x^*(\delta^*)], \\ & \text{subject to } x^*(\delta^*) \text{ satisfying (7).} \end{aligned}$$

- $S^0$  and  $\delta^*$  are such that,

$$\bar{x}(Q^H; I^0; \delta^*) < \bar{x}(S^0; I^0; \delta^*). \quad (9)$$

Further, there exists a  $\omega^0$  such that if  $\omega' \geq \omega^0$ , then the solution to problem (8) satisfies inequality (9).

Proposition 1 demonstrates that there exist parameterizations of the model such that Condition 1 holds when all players play according to the proposed strategies. For Condition 1 to hold,  $S^0$  must be high enough such that the marginal collector who bids in stage 2 would rather bid in stage 2 upon observing a bid in stage 1 and realizing that he will pay  $Q^H$  upon winning the auction than not observing a bid in stage 1 and paying  $S^0$  upon winning. Equation (8) makes explicit the relationship between  $S^0$  and  $\omega'$ :

the higher is  $\omega'$ , the higher will be  $S^0$ . Thus, for Condition 1 to hold requires  $\omega'$  to be sufficiently large. If the uninformed seller's choice of starting price under the parameterization is globally optimal, then said strategies make up an ISBE.

It should be noted that there exists a solution to equation (8) in which the constraint that  $S^0 \leq 1$  binds for  $\omega'$  sufficiently large. In such an equilibrium, all dealers are indifferent between not bidding at all and bidding in accordance with the equilibrium strategy. However, as long as  $\delta$  is such that inequality (4) holds, then bids by dealers are informative to collectors even if they have no effect on the end price. In such case,  $x^*(\delta) \leq 1$  and there exist collector types such that  $x < x^*(\delta)$  that would bid in stage 2 only upon observing a bid in stage 1.<sup>13</sup>

## 4 Characterization of All Equilibria

The previous section restricted attention to establishing those parameterizations of the model that give rise to an ISBE. This section serves to characterize all equilibria of the game. In the previous section, Condition 1 demonstrated that an ISBE requires that collectors be positively influenced by stage 1 bidding, so much so that they prefer to pay  $Q^H$  upon observing a stage 1 bid over paying  $S$ , an amount less than  $Q^H$ , upon not observing one. Therefore, by relaxing Condition 1, I establish alternate equilibria in which collectors are not inclined to increase their bids upon observing stage 1 bidding. Additionally, Proposition 1 required that  $S$  be no greater than  $Q^H$  in an ISBE. Therefore, I relax that constraint in this section and establish a third type of equilibrium in which shill bidding is precluded due to the uninformed seller setting a starting price above what a dealer would be willing to bid. The section concludes by demonstrating that the three classes of equilibria are comprehensive of all equilibria, thereby allowing for a characterization of the uninformed seller's optimal starting price.

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<sup>13</sup>The case in which  $S \geq 1$  and  $\delta = 0$  is considered in the following section.

## 4.1 Additional Subgame Equilibria

I begin by characterizing an equilibrium of the game in which inequality (9) does not hold. The intuition for such an equilibrium is established by considering possible deviations from the ISBE strategies. When inequality (9) is reversed, assuming all other players continue playing according to their ISBE strategies, a collector may find a profitable deviation as follows: collector type  $\bar{x}(Q^H; I^0; \delta)$ , who did not bid unless a bid in stage 1 was observed, can now profitably deviate by bidding in stage 2 when no bid is observed in stage 1 and not bid when one is. Working backward to such a buyer's stage 1 bid, it is evident that he can place a bid of  $Q^H$  in stage 1 that has him pay  $Q^H$  if a bid of  $Q^H$  is placed by an opposing bidder (either a dealer or a fraudulent seller), or  $S$  if no bids are placed by opposing bidders. By construction of  $\bar{x}(Q^H; I^0; \delta)$ , either outcome leaves him no worse off than under the ISBE strategy when inequality (9) is reversed.

Before establishing the aforementioned bidding strategy as a best response under an alternate equilibrium, it is necessary to introduce some notation. Suppose that in the equilibrium under consideration, dealers bid in stage 1 with probability  $\delta$  and fraudulent sellers shill in stage 1 with probability  $\sigma$ . Conditional upon bidding in stage 1, a dealer will bid  $Q^H$  due to arguments used in the proof of Lemma 1. Under the restriction that a fraudulent seller cannot take any action that would have him be identified as a fraudulent seller, the fraudulent seller must only bid  $Q^H$ . As before, let  $I^0$  and  $I'$  denote a collector's information sets upon observing and not observing a bid in stage 1, respectively. A collector's beliefs in information sets  $I^0$  and  $I'$  are expressed as follows:

$$\mu\{I^0; \delta, \sigma\} = \frac{\gamma\delta\mu\{I_1\}}{\gamma\delta\mu\{I_1\} + \phi\sigma(1 - \mu\{I_1\})}, \quad (10)$$

and

$$\mu\{I'; \delta, \sigma\} = \frac{(1 - \gamma\delta)\mu\{I_1\}}{(1 - \gamma\delta)\mu\{I_1\} + (1 - \phi\sigma)(1 - \mu\{I_1\})}. \quad (11)$$

As a last bit of notation, let  $\bar{x}(Q^H; I^0; \delta, \sigma)$  and  $\bar{x}(S; I'; \delta, \sigma)$  be defined as before, only now making explicit the reliance of collectors' beliefs on  $\sigma$  as well as  $\delta$ . The collector's best response can now be described as follows.

**Lemma 4** *In any equilibrium such that  $\bar{x}(S; I'; \delta, \sigma) < \bar{x}(Q^H; I^0; \delta, \sigma)$ , a collector's best response,  $b^*$ , calls for him to bid only in stage 1 as follows:*

- If  $x \geq \bar{x}(Q^H; I^0; \delta, \sigma)$ , bid  $v(x; I^0; \delta, \sigma)$ ;
- If  $x \in [\bar{x}(S; I'; \delta, \sigma), \bar{x}(Q^H; I^0; \delta, \sigma))$ , bid  $S$ ;
- Otherwise, do not bid.

The collector's best response,  $b^*$ , as characterized by Lemma 4, separates collectors between those that prefer to win the auction when the price is as high as  $Q^H$  from those who only prefer to win the auction when the price is below  $Q^H$ . In contrast to  $b^0$ , a collector's strategy under the ISBE, bids are placed in stage 1 only. In this way, collectors do not respond to the bidding of dealers. Consequently, there is no incentive for a dealer to not bid in stage 1, which implies  $\delta$  will be equal to unity in equilibrium.

To characterize the fraudulent seller's bidding strategy, let  $R^I(Q^H; b^*, 1, \sigma)$  and  $R^I(0; b^*, 1, \sigma)$  denote the fraudulent seller's revenue from shilling and not shilling respectively when collectors bid according to  $b^*$ , dealers are believed to bid with probability one, and when fraudulent seller's are believed to shill with probability  $\sigma$ . A necessary condition for an equilibrium in which  $\sigma = 1$  is,

$$R^I(Q^H; b^*, 1, 1) \geq R^I(0; b^*, 1, 1). \quad (12)$$

If inequality (12) does not hold, then  $\sigma = 1$  is not in equilibrium. However,  $\sigma = 0$  is unlikely to be part of an equilibrium either, since if it were known that all fraudulent sellers refrain from shilling, then upon observing a bid in stage 1, collectors would

assign a probability of one that the item were genuine. This argument is evident from setting  $\sigma$  to zero in equation (10). In that case, placing a skill bid of  $Q^H$  in stage 1 is likely a profitable deviation for the fraudulent seller. Thus, it would seem that the fraudulent seller's strategy should be purely mixed when (12) does not hold.

The following proposition characterizes a *non-informational skill bidding equilibrium* ("Non-ISBE") in which the uninformed seller's starting price,  $S^*$ , as well as collectors' beliefs as dictated by  $\delta$  and  $\sigma$ , causes inequality (9) to be reversed, thereby making stage 1 bids uninformative.

**Proposition 2 Non-ISBE.** *If the seller's optimal starting price gives rise to the following, then the following constitute an equilibrium:*

- Dealers bid  $Q^H$  in stage 1 with probability one.
- Collectors bid according to strategy  $b^*$ .
- Fraudulent sellers bid  $Q^H$  in stage 1 with probability  $\sigma^*$ , equal to: unity if

$$R^I(Q^H; b^*, 1, 1) \geq R^I(0; b^*, 1, 1);$$

or some value in  $[0, 1)$  satisfying

$$R^I(Q^H; b^*, 1, \sigma^*) = R^I(0; b^*, 1, \sigma^*)$$

otherwise.

- Uninformed sellers and fraudulent sellers set a starting price of  $S^*$  solving:

$$\begin{aligned} \max_{S \in [0,1]} \gamma \mu_0 \left( [1 - \bar{x}(Q^H; I^0; 1, \sigma^*)] + S \bar{x}(Q^H; I^0; 1, \sigma^*) \right) & \quad (13) \\ + (1 - \gamma \mu_0) (S [1 - \bar{x}(S; I'; 1, \sigma^*)] + \omega' \bar{x}(S; I'; 1, \sigma^*)) & \end{aligned}$$

and

- $S^*$  and  $\sigma^*$  are such that:

$$\bar{x}(Q^H; I^0; 1, \sigma^*) \geq \bar{x}(S^*; I'; 1, \sigma^*). \quad (14)$$

Further, there exists a  $\omega^*$  such that if  $\omega' \leq \omega^*$ , then the solution to problem (13) satisfies inequality (14).

Proposition 2 establishes a Non-ISBE such that if shill bidding occurs, it is uninformative to collectors. Since stage 1 bidding is uninformative to them, a consequence of inequality (14), collectors bid only in stage 1. Collectors' stage 1 bids are such that they take into account that the seller may be a fraudulent seller, based on their beliefs from having reached information set  $I_1$ . Since collectors do not bid in response to stage 1 bidding, dealers necessarily bid in stage 1. Fraudulent sellers however, cannot use shill bidding to deceive collectors into increasing their bids in stage 2. Therefore, they shill only as a *competitive shill*—when they would have otherwise preferred a starting price of  $Q^H$  to a starting price of  $S^*$ . The uninformed seller sets a starting price of  $S^*$ , which is mimicked by a fraudulent seller, that maximizes the seller's expected surplus, given the strategies of others within the equilibrium characterization.

Having characterized a Non-ISBE, I consider possible equilibria in which  $S \geq 1$ . The characterization of an ISBE constrained the uninformed seller's starting price to be no greater than 1. This is because if  $S > 1$ , dealers would find it unprofitable to bid, thus precluding fraudulent sellers from placing shill bids. However, as was discussed in the previous section, there may ISBE in which  $S^0 = 1$  and  $\delta^* > 0$ , so that dealers still bid in stage 1 with positive probability even if their bids confer no benefit to them. In this case, informational shill bidding is beneficial to a fraudulent seller as long as inequality (9) holds. My purpose in this section is to characterize equilibria in which  $\delta^* = 0$ .

Without the potential to learn from stage 1 bidding, the best response of collectors must have them bid only in stage 1, similar to  $b^*$ . Unlike  $b^*$ , however, which takes into account the collector's valuation ex-post, when all dealers are screened out due to the high starting price, the collector's beliefs post-stage 1 are identical to their beliefs pre-stage 1,  $\mu \{I_1\}$ . In this way, define an indifferent collector type,  $\bar{x}(S; I_1) = \min \{x \mid v(x; I_1) = S\}$ . The following proposition characterizes a *no skill bidding equilibrium* ("NSBE") such that  $S \geq 1$ .

**Proposition 3 NSBE.** *If the seller's optimal starting price is no less than 1, then the following constitute an equilibrium:*

- *Collector's bid according to  $b'$ , characterized by the following: if  $x \geq \bar{x}(S'; I_1)$ , bid  $v(x; I_1)$  in stage 1; otherwise, do not bid.*
- *Neither dealers nor fraudulent sellers bid in either stage.*
- *Both uninformed sellers and fraudulent sellers set a starting price of  $S'$  solving:*

$$\max_{S \geq 1} S [1 - \bar{x}(S; I_1)] + \omega(\mu_0) \bar{x}(S; I_1). \quad (15)$$

*Further, the constraint that  $S \geq 1$  does not bind on the solution to problem (15) if the following holds:*

$$\omega(\mu_0) > 2(1 - \mu \{I_1\}).$$

As a final case, consider potential equilibria in which dealers bids only in stage 2. Proposition 1 established that in the ISBE, dealers may bid in stage 2 with positive probability. However, it was argued that if dealers only bid in stage 2 and never in stage 1, then inequality (9) would not hold. Therefore, it seems natural to ask whether there exists an equilibrium in which a dealer bids only in stage 2 regardless of whether inequality (9) holds. Proposition 4 answers negatively.



**Proposition 4** *There does not exist an equilibrium of the game in which dealers bid only in stage 2.*

The reason why there cannot be an equilibrium in which dealers bid in stage 2 is that there does not exist a set of beliefs such that neither a dealer nor fraudulent seller finds it profitable to deviate by bidding in stage 1. To see this, first recognize that were dealers and fraudulent sellers to bid only in stage 2, collectors would bid according to a strategy similar  $b^*$ .<sup>14</sup> Since a bid in stage 2 subjects a dealer to the risk of transmission failure, to rationalize stage 2 bidding, the beliefs of collectors upon observing a bid in stage 1 must be such that they reduce the dealer's expected profit. But for that to happen, collectors would have to increase their assessment of the item following observing a stage 1 bid to the point that certain collector types that had bid  $S$  in stage 1, would increase their bid upon observing a bid in stage 1 to at least  $Q^H$  in stage 2. In that case, a fraudulent seller could profitably deviate by bidding in stage 1.

## 4.2 Optimal Starting Price

In characterizing the uninformed seller's optimal starting price, it is necessary to first characterize the subgame equilibrium following each possible choice of starting price. The following result indicates that the three classes of subgame equilibria are comprehensive of all equilibria in the game.

**Proposition 5** *Consider an uninformed seller's choice of  $S$ . If  $S \leq 1$ , then for any strategy played by dealers and fraudulent sellers, the resulting subgame equilibrium is one of two types: either  $\bar{x}(Q^H; I^0; \cdot) < \bar{x}(S; I'; \cdot)$  or  $\bar{x}(Q^H; I^0; \cdot) \geq \bar{x}(S; I'; \cdot)$ . If the former, then  $b^0, \sigma^* = 1$  and  $\delta^*$  as characterized in Proposition 1 constitute a subgame*

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<sup>14</sup>The bidding strategy would be modified to account for the probability that stage 2 bids by dealers or by fraudulent sellers may not be transmitted.

*equilibrium following from  $S$ . If the latter, then  $b^*$ ,  $\delta^* = 1$  and  $\sigma^*$  as characterized by Proposition 2 constitute a subgame equilibrium following from  $S$ . If  $S > 1$ , then  $b'$ ,  $\delta^* = \sigma^* = 0$  constitute the only subgame equilibrium following from  $S$ .*

Proposition 5 demonstrates that the three classes of equilibria are comprehensive. The key insight behind the result is that for a given  $S$ , the resulting subgame equilibrium is predicated on the beliefs of collectors, of which  $\delta^*$  and  $\sigma^*$  are key inputs. First, it is demonstrated that for any set of beliefs collectors may have, a dealer's best response necessarily involves him bidding  $Q^H$  in stage 1 with probability  $\delta$ . Upon inputting  $\delta^*$  and  $\sigma^*$  into their beliefs, collectors bid according to  $b^0$  or  $b^*$  depending on the direction of the inequality between  $\bar{x}(Q^H; I^0; \cdot)$  and  $\bar{x}(S; I'; \cdot)$ . Since collectors' beliefs are assumed to be common knowledge, dealers and fraudulent sellers understand whether  $b^0$  or  $b^*$  is a best response. Whether  $b^0$  or  $b^*$  is the best response, dealers and fraudulent sellers choose whether or not to bid in stage 1. The restriction on collectors' beliefs is that they must be *correct* in predicting the bidding behavior of dealers and fraudulent sellers. Given that collectors' beliefs may be self-fulfilling, it follows that for a given  $S$ , both the ISBE and Non-ISBE subgames may be supported. Further, when  $S = 1$ ,  $\delta^* > 0$  can support an ISBE subgame equilibrium and  $\delta^* = 0$  supports a NSBE subgame equilibrium.

Understanding the beliefs, and consequently the subgame equilibrium associated with each choice of  $S$ , the uninformed seller's optimal starting price maximizes his surplus over all values of  $S$ . Proposition 2 showed that for small values of  $\omega'$ , which itself is a function of the uninformed seller's idiosyncratic preference and his beliefs, the solution problem (13) gives rise to a starting price that is consistent with the Non-ISBE subgame. Proposition 1 showed that for large values of  $\omega'$ , the solution to problem (8) gives rise to a starting price that is consistent with the ISBE subgame. As the uninformed seller's valuation becomes larger still, the optimal starting price may be either a value

of  $S = 1$ , which is consistent with the ISBE subgame or a value of  $S > 1$ , consistent with the NSBE subgame. The comparison between the surplus earned from these two choices is determined by whether  $\delta^* > 0$  when  $S = 1$ . If  $\delta^* = 0$  at  $S = 1$ , then the ISBE and NSBE subgames are equivalent since neither involve stage 1 bids from dealers or fraudulent sellers. But when  $\delta^* > 0$  at  $S = 1$ , the uninformed seller's surplus is increased by the fact that bids by dealers can give rise to collectors bidding in stage 2, whereas they would have otherwise not bid at all. Further, as long as inequality (9) holds, the overall effect on the uninformed seller's surplus is unambiguously positive. Therefore, if  $\delta^* > 0$  at  $S = 1$ , then there exists some  $\varepsilon > 0$  such that if the uninformed seller's solution in a NSBE subgame is  $S' = 1 + \varepsilon$ , the uninformed seller's surplus is higher by setting a starting price of  $S^* = 1$ .

In general, the number of parameters to consider make drawing general conclusions regarding the optimal starting price and the resulting equilibrium difficult. The following section considers a parametric example to demonstrate the effect of the seller's valuation, the beliefs of uninformed sellers and buyers and the proportion of informed sellers in the market affect the equilibrium.

## 5 Numerical Example

The previous two sections established three possible classes of equilibria, which were shown to be exhaustive of all equilibria of the game. The purpose of this section is to better understand the conditions under which each equilibrium occurs, and see how changes in certain parameter values affect surplus within a numerical example.

The results of this section rely on a number of parameterizations, which are discussed in what follows. First, let  $\alpha = .6$ . The proof of Lemma 3 showed that inequality (4) holds for any  $\alpha \geq .5$ . The choice of  $\alpha = .6$  ensures that this condition holds for any

$S \leq 1$ .<sup>15</sup> Next, let  $\gamma = .1$  and let  $\phi$  vary between zero and .1. It seems reasonable to assume that both  $\gamma$  and  $\phi$  are small so that the presence of a dealer or a fraudulent seller is considered a rare event. The assumption that  $\phi$  is less than  $\gamma$  is a necessary condition for inequality (9) to hold.<sup>16</sup> Further, I make the following functional form assumption that  $\omega(\mu\{I\}) \equiv \omega \cdot \mu\{I\}$ . Under this functional form, it follows that:

$$\omega' = \omega \cdot \frac{(1 - \gamma) \mu_0}{1 - \gamma \mu_0^2}$$

and

$$\omega(\mu_0) = \omega \cdot \mu_0.$$

Throughout this section,  $\omega$  and  $\mu_0$  are varied between zero and one.

Figure 3.1 illustrates the values of  $\mu_0$  and  $\omega$  giving rise to each class of subgame equilibrium, keeping  $\phi$  constant. The area above curve A represents the  $\{\mu_0, \omega\}$  combinations for which inequality (9) holds. The area below curve B represents the  $\{\mu_0, \omega\}$  combinations for which inequality (14) holds. Thus  $\{\mu_0, \omega\}$  combinations between curves A and B can give rise to an ISBE or a Non-ISBE depending upon the beliefs held by collectors over  $\delta$  and  $\sigma$ . These beliefs then govern whether the starting price chosen by uninformed sellers and mimicked by fraudulent sellers is a solution to problem (8) (giving rise to an ISBE) or a solution to problem (13) (giving rise to a Non-ISBE). Interestingly, along all points between curves A and B, collectors' surplus (integrating across all collector types) as well as total surplus is higher in the Non-ISBE than the ISBE while the uninformed seller's surplus is higher in the ISBE.

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<sup>15</sup>It is assumed that  $\delta^* = 1$  even when  $S = 1$ .

<sup>16</sup>The second-order condition, by which the interior solution to problem (8) is optimal requires  $\gamma$  to be sufficiently small.

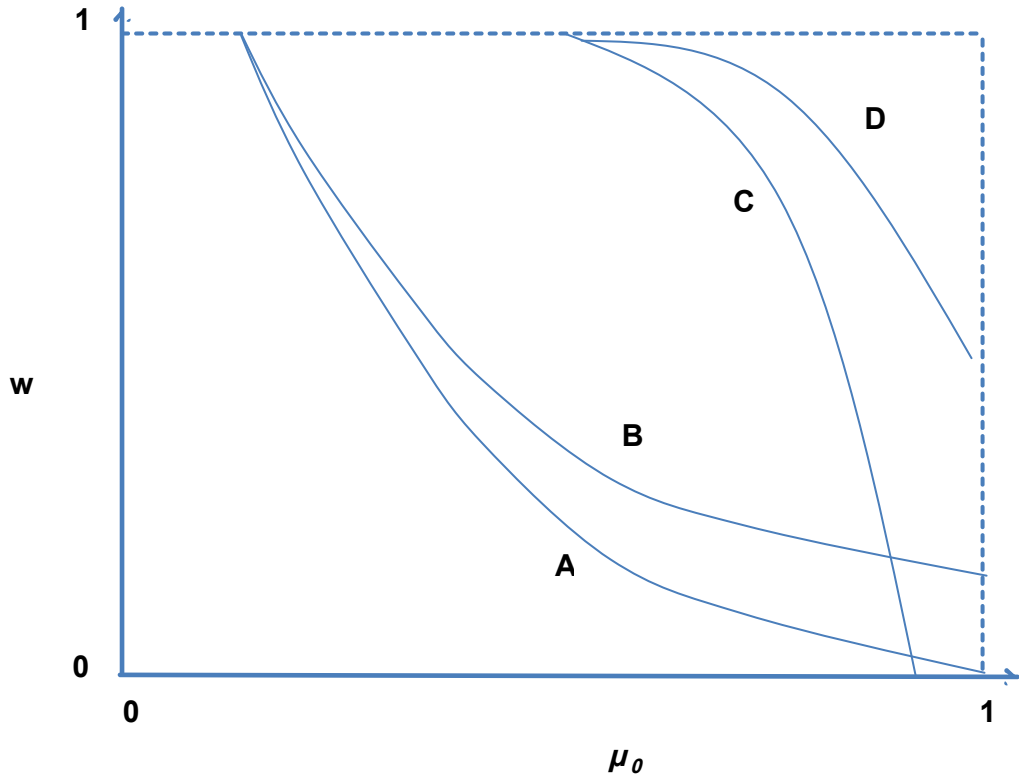


Figure 3.1: Equilibrium characterization in  $\{\mu_0, \omega\}$  space.

The area above curve C reflects combinations of  $\{\mu_0, \omega\}$  for which  $S^0 = 1$ . The area above curve D, not inclusive of the curve itself, reflects  $\{\mu_0, \omega\}$  combinations such that  $S' > 1$ . Thus for all points above and inclusive of curve C, the beliefs of collectors are in a sense dictated by  $S$ . If  $S = 1$ , collectors may believe that  $\delta > 0$ , thus giving rise to a ISBE in which  $S^0 = 1$ . Not surprisingly, for all points between curves C and D, and for any  $\delta > 0$ , the uninformed seller's surplus is higher in an ISBE in which  $S^0 = 1$  than in a NSBE in which  $S' = 1$ . Moving further to the northeast beyond curve D, eventually the uninformed seller's surplus is higher upon choosing  $S > 1$  and having buyers bid in accordance with the NSBE than in choosing  $S = 1$  and having them bid in accordance with the ISBE. The reason for this is that at lower levels of  $\mu_0$ , observing a bid in stage 1 is very informative to collectors. However, at higher values of  $\mu_0$ , collectors' valuations

are high enough that observing a bid in stage 1 has little effect. The uninformed seller prefers to exploit the high valuations of collectors by setting a starting price in excess of 1 rather than setting a starting price no greater than 1 and have to rely on the presence of dealers to convince collector's to raise their bids.

The equilibrium starting price path for a given path is described in reference to Figure 3.1 as follows: For points near the axis, the equilibrium starting price solves problem (13). Moving to the northeast while staying below curve A,  $S^*$  is increasing in  $\mu_0$  and  $\omega$  so the equilibrium starting price is increasing. For all points between curves A and B,  $S^0 > S^*$ . Therefore, the optimal starting price will have a jump point once the value in  $\mu_0$  and also in  $\omega$  is reached such that the equilibrium switches from a Non-ISBE to an ISBE. Moving further in the northeast direction to points between curves B and D, the optimal starting price is increasing as  $S^0$  is increasing in both  $\mu_0$  and  $\omega$ . The optimal starting price will again reach a jump point beyond curve D when the equilibrium switches from an ISBE in which  $S^0 = 1$  to a NSBE in which  $S' > 1$ . This last point requires that  $\delta^* > 0$  in the ISBE in which  $S^0 = 1$ ; otherwise, the jump point disappears.

The effect of a change in  $\phi$  on the equilibrium characterization can also be seen in reference to Figure 3.1. As  $\phi$  increases, both curves A and B rotate upward from their common pivot point along the  $\omega = 1$  line. Likewise, curves C and D rotate upward from their common pivot point along the  $\omega = 1$  line. In doing so, this the set of  $\{\mu_0, \omega\}$  combinations giving rise to an ISBE and NSBE shrink and the set of combinations giving rise to a Non-ISBE increases. The opposite is true as  $\phi$  decreases; however, the fact that the axis of rotation (along the horizontal dotted line at  $\omega = 1$ ) does not move indicates that there is a threshold value of  $\mu_0$  such that an ISBE is precluded for any  $\mu_0$  below the threshold.<sup>17</sup> Likewise, there is a threshold value of  $\mu_0$ , as shown by the pivot point of D along the  $\omega = 1$  line, below which a starting price strictly exceeding 1 is precluded. On the other hand, for  $\phi = \gamma$  the entire range of  $\{\mu_0, \omega\}$  values gives rise to

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<sup>17</sup>This is true given the functional form assumption on  $\omega(\cdot)$ , but needn't be true in general.

a Non-ISBE as do values of  $\phi > \gamma$ , while the ISBE is precluded for all  $\phi \geq \gamma$ .

Further, as  $\phi$  increases, so do the set of  $\{\mu_0, \omega\}$  points in which  $\sigma^* < 1$ . For a given  $\phi$ , inequality (12) fails to hold only at points nearest the axes. As  $\phi$  increases and curve B rotates upward, the  $\{\mu_0, \omega\}$  combinations for which inequality (12) fails to hold expand outward. However, for the parameterizations considered, inequality (12) still holds for all  $\{\mu_0, \omega\}$  combinations above curve A for all values of  $\phi$ .

Consider now the total surplus within the ISBE and Non-ISBE, respectively. Total surplus in both types of equilibria behaves similarly across changes in  $\mu_0$ ,  $\omega$ , and  $\phi$  in that it is:

- increasing in  $\mu_0$ ;
- decreasing in  $\omega$  at low values of  $\omega$ , then increasing at higher values; and
- decreasing in  $\phi$ .

The increase in surplus due to an increase in  $\mu_0$  results from the simple fact that the object for sale is more likely to be genuine. The ambiguity of surplus with respect to  $\omega$  is due to the effect of  $\omega$  on the choice of starting prices  $S^0$  and  $S^*$ , respectively. An increase in  $\omega$  leaves an uninformative seller with higher surplus should the auction fail to result in a sale. At the same time, it causes the equilibrium starting price to increase, which reduces the probability of a sale to a buyer with a potentially higher valuation. Thus, it is not obvious whether an increase in  $\omega$  should lead to an increase or decrease in total surplus. With respect to  $\phi$ , an increase in its value means that the seller is more likely to be fraudulent and hence the item is more likely to be a fake.

It is interesting to note that of all the  $\{\mu_0, \omega, \phi\}$  combinations giving rise to both the ISBE and the Non-ISBE, the Non-ISBE results in greater surplus than the ISBE. This gives rise to a counter-intuitive result that while surplus in either equilibrium decreases with an increase in  $\phi$ , an increase in  $\phi$  can still increase surplus. To see this, recall that

as  $\phi$  increases, curves A and B rotate upward. Thus, the set of  $\{\mu_0, \omega\}$  combinations giving rise to an ISBE shrinks while the set giving rise to a Non-ISBE gets larger. Thus for values of  $\{\mu_0, \omega\}$  along curve A, an increase in  $\phi$  results in players switching from an ISBE to a Non-ISBE. It can be shown that for a sufficiently small increase in  $\phi$ , the resulting Non-ISBE yields greater surplus than the ISBE that would have been played at the smaller value of  $\phi$ .

## 6 Conclusion

This paper was motivated by the desire to understand from a game-theoretic standpoint, the economic conditions and behavior giving rise to the type of fraud illustrated by the Diebenkorn example. A central achievement of this paper has been to show that such fraud can be perpetrated upon rational buyers behaving in accordance with equilibrium strategies. This is the upshot of Proposition 1 in establishing the *informational skill bidding equilibrium* (ISBE). In doing so, the characteristics of the marketplace and of the auction mechanism itself that allow for such an equilibrium to persist were illustrated. Firstly, the proportion of fraudulent sellers (those entering the market with the purpose of defrauding buyers) must make up a sufficiently small proportion of total sellers. If the proportion of fraudulent sellers is too high, uninformed buyers will fail to be swayed by the bidding of others and the fraud cannot be carried out. Further, when bidding by other players serves as a positive indication of the item's quality, uninformed sellers (those with honest intentions but imperfect information) must set a starting price high enough that paying the starting price in an auction that did not involve a competing bid is less attractive than paying a higher price in an auction that did. When this condition holds, a certain set of uninformed buyers will enter the bidding upon observing a bid from a competing buyer. At the same time, the starting price cannot be so high as to screen out the bids of informed buyers.



In order to characterize all equilibria of the game, I sought to establish an equilibrium when uninformed buyers are not influenced by the bidding of others. I therefore established a *non-informational shilling bidding equilibrium* (Non-ISBE), in which the proportion of fraudulent sellers is sufficiently high that upon observing a bid from another buyer, uninformed buyers are not compelled to increase their previous bid. Knowing that they can't very well lower their bid upon observing a bid from an opposing buyer, uninformed buyers bid only once but take this information into account in determining the optimal bid amount. I also established a *no shill bidding equilibrium* (NSBE) in which the uninformed seller finds it profitable to screen out all informed buyers by announcing a starting price above what such buyers would be willing to pay. The three classes of equilibria are shown to be comprehensive of all equilibria of the game.

In examining the parameterizations giving rise to the three classes of equilibria, it is evident that for the range of parameter values considered, the Non-ISBE is essentially the counter to the ISBE. While the ISBE requires high initial values amongst uninformed sellers and uninformed buyers, the Non-ISBE requires low values. The NSBE requires initial values that are higher still than those giving rise to an ISBE. The restriction on initial valuations giving rise to the ISBE becomes more strict as the proportion of fraudulent sellers in the market increases. There is however, a small sub-set of the parameter space giving rise to both the ISBE and Non-ISBE. Holding constant all parameters within this space, the Non-ISBE always results in higher total surplus than the ISBE. Thus, an increase in the proportion of fraudulent sellers may actually serve to increase total surplus if it causes players to switch from the ISBE to the Non-ISBE. This result asserts an additional level of complication to be considered by a mechanism designer in designing the auction and enacting policies for the detection and prosecution of fraud. That is to say, it is not enough to consider what changes in certain parameters may affect the efficiency of the mechanism across a given equilibrium characterization as changes in parameters may give rise to entirely new equilibria. The consideration of

this mechanism design question is left for future work.

# APPENDIX

## A Proofs

### A.1 Proof of Lemma 1

The proof of both parts follows from standard auction-theoretic arguments where a buyer knows his valuation but not the valuation of others. I formalize the first part of the lemma, then extend the result to the second. In doing so, consider a collector who finds himself at information set  $I$  following the stage 1 bidding in which the standing price is  $S_1$ . His valuation upon obtaining the item following stage 2 is  $v(x; I)$ . Note that since all the information that can be gleaned from other participants would have been revealed in stage 1, the buyers unconditional expected value and his expected value, conditional upon obtaining the item are equal. If his stage 1 bid exceeds  $v(x; I)$ , then any further bids in stage 2 can only increase the probability of his winning the auction and receiving negative surplus. Thus, he is better off not bidding in stage 2. Now suppose his stage 1 bid was less than  $v(x; I)$ . Further, let  $Y$  denote the maximum of  $S_1$  and the highest bid of all opposing bidders in stage 2 and let  $V$  denote his expected surplus if his stage 2 bid does not transmit. Clearly,  $V$  is positive if the bidder in question is the high bidder following stage 1, zero otherwise, and is independent of his stage 2 bid. Bidding some amount  $B$  in stage 2, if transmitted, has him obtain the item at  $Y$  if  $Y \leq B$  and lose out if  $Y > B$ . He then chooses  $B^*$  to solve:

$$\max_{B \geq S_1} (1 - \alpha) \int_{S_1}^B (v(x; I) - Y) dY + V.$$

This equation is solved by  $B^* = v(x; I)$  if  $v(x; I) \geq S_1$ .

The second part of the lemma is proven trivially by substituting  $Q^H$  for  $v(x; I)$ .

## A.2 Proof of Lemma 2

In light of Lemma 1, there are three classes of collectors' strategies to consider: bid  $v(x; I^0)$  in stage 1; bid some amount  $b$ , less than  $v(x; I^0)$ , in stage 1 and bid  $v(x; I)$  in stage 2 if  $b < v(x; I)$ , for any information set  $I$  reached following stage 1; and do not bid in stage 1 and bid  $v(x; I)$  in stage 2, for any  $I$  reached following stage 1.

Working backward, I begin by considering the payoff to bidding  $v(x; I^0)$  – assumed greater than  $\beta$  – in stage 2, conditional upon  $I^0$  being reached in stage 1. If his bid is transmitted, he obtains the item and pays  $Q^H$  (normalized to unity) if the dealer's bid transmits,  $\beta$  if it does not; if his bid is not transmitted, he receives nothing. The expected payoff is expressed as a function of  $T$ , a random variable, equal to 1 if his opponent's stage 2 bid transmits, 0 otherwise. His expected payoff is then:

$$\Pi^C(0; x, \beta, \sigma) = (1 - \alpha) P\{I^0\} [\mu\{I^0\} (1 + x) - P\{T = 0\} \beta - P\{T = 1\}]$$

Within the notation  $\Pi^C(0; x, \beta, \sigma)$ , the first term reflects that he bids zero in stage 1 and optimally in what follows. The second term is his preference parameter. The third term reflects that his decision is based upon knowing that dealers and fraudulent sellers bid  $\beta$  in stage 1. His payment upon winning the auction depends on whether or not the opposing bidder's stage 2 bid is transmitted. He pays  $Q^H$  if it is transmitted,  $\beta$  if it isn't. If the opposing bidder is a dealer, his bid is transmitted with probability  $\alpha$ . If the opposing bidder is a shilling seller, then let  $\sigma$  denote the probability that his stage 2 bid is transmitting upon allowing for the seller to play a (possibly) mixed strategy in stage 2.<sup>18</sup>

Consider then the payoff to bidding  $v(x; I^0)$  in stage 1. Since a bid of  $v(x; I^0)$  will be at least as great as his valuation following the stage 1 bidding, there is no circum-

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<sup>18</sup>The fraudulent seller does not violate the condition that he be undetectable as a fraudulent seller by not mimicking the dealer's stage 2 bid since not all stage 2 bids are transmitted.

stance under which he will bid in stage 2. By bidding  $v(x; I^0)$  – assumed greater than  $\beta$  – in stage 1, then should  $I^0$  be reached, he receives the same payoff as he would have bidding  $v(x; I^0)$  in stage 2, only his payoff is not subject to transmission error. Should  $I'$  be reached, then he obtains the item and pays  $S$ . His expected payoff can be expressed as:

$$\Pi^C(v(x; I^0); x, \beta, \sigma) = \Pi^C(0; x, \beta, \sigma) / (1 - \alpha) + P\{I'\} [\mu\{I'\} (1 + x) Q^H - S] .$$

Define  $x_\beta^*$  as the value of  $x$  such that

$$\Pi^C(0; x_\beta^*, \beta, \sigma) = \Pi^C(v(x; I^0); x_\beta^*, \beta, \sigma) .$$

If such a point exists, then the fact that

$$\frac{\partial \Pi^C(v(x; I^0); x, \beta, \sigma)}{\partial x} > \frac{\partial \Pi^C(0; x, \beta, \sigma)}{\partial x}$$

implies that any collector such that  $x > x_\beta^*$  strictly prefers to bid  $v(x; I^0)$  in stage 1. Under Condition 1, all collector types prefer to obtain the item in information set  $I^0$  than in  $I'$ . Notice that in order for the indifference condition to hold, it must be the case that

$$\mu\{I'\} (1 + x) Q^H - S < 0$$

for values of  $x \leq x_\beta^*$ ; otherwise all collector types prefer to bid  $v(Q^H; I^0)$  in stage 1. It follows from Condition 1 that:

$$x_\beta^* \in (\bar{x}(Q^H; I^0), \bar{x}(S; I')) .$$

Thus, for any collector such that  $x < x_\beta^*$ , it is not profitable to bid in stage 2 upon finding

himself in  $I'$  following stage 1.

It remains to show that no bidding strategy that has a collector bid in both stages yields as high a surplus as  $\Pi^C(v(x; I^0); \beta, \sigma)$ . Consider this in two parts. First, consider a strategy of bidding some amount  $b > \beta$  in stage 1 and optimally in stage 2. Bidding  $b$  instead of  $v(x; I^0)$  in stage 1, has no effect on the buyer's payoff should  $I'$  be reached; but if  $I^0$  is reached, bidding  $b$  affects his payoff by causing him to win the auction with a lower probability. For any collector such that  $x > \bar{x}(Q^H; I^0)$ , this yields a lower expected payoff. Next, consider the possible strategy of bidding  $b < \beta$  in stage 1. As compared to a bid of  $v(x; I^0)$ , such a bid has no effect on the collector's payoff should  $I'$  be realized. If  $I^0$  is realized, such a strategy introduces the possibility that he lose out on a profitable trade if his stage 2 bid is not transmitted. As compared to not bidding at all in stage 1, bidding  $b < \beta$  gives the collector the same payoff should  $I^0$  be reached, but gives him negative surplus should  $I'$  be reached, where as not bidding at all would have given him zero surplus. Thus, no collector will ever bid  $b < \beta$ .

### A.3 Proof of Lemma 3

The proof begins by showing that conditional on bidding some amount  $\beta$  in stage 1, the only value of  $\beta$  that can be part of an ISBE is  $Q^H$ . When collector's bid according to  $b_\beta^0$ , the magnitude of the dealer's bid has no effect on the collector's stage 2 action: if  $x < x_\beta^*$ , then any bid in  $[S, Q^H]$  will result in a standing price of  $S$  following stage 1, so no deviation will be detected; if  $x \geq x_\beta^*$ , then any deviation from  $\beta$  will be detected by the collector following stage 1, but it will be too late for him to change his bidding in response. Since collectors do not respond to the dealer's bid, the standard arguments regarding second price auctions support  $Q^H$  as the only equilibrium candidate.

Given the previous result, let  $x^* \equiv x_{Q^H}^*$  and  $b^0 \equiv b_{Q^H}^0$ . To show the conditions under which a dealer would bid in stage 1, compare the dealer's expected surplus from bidding

$Q^H$  in stage 1 when collectors bid according to  $b^0$ ,  $\Pi^D(Q^H; b^0)$ , to his expected surplus from bidding  $Q^H$  in stage 2,  $\Pi^D(0; b^0)$ . It follows that,

$$\Pi^D(Q^H; b^0) = (1 - S) [P\{x < \bar{x}(Q^H; I^0)\} + \alpha P\{x \in [\bar{x}(Q^H; I^0), x^*]\}] .$$

This expression follows from the fact that the dealer only obtains the item when the collector does not bid, in which case he pays  $S$ . The collector does not bid if either  $x < \bar{x}(Q^H; I^0)$  or if  $x \in [\bar{x}(Q^H; I^0), x^*]$  and his stage 2 bid fails to transmit.

In considering waiting for stage 2 to bid, the dealer gains an informational advantage. By not bidding in stage 1, the dealer puts a collector with  $x \in [\bar{x}(Q^H; I^0), x^*]$  into information set  $I'$ , causing him not to bid in stage 2 when he otherwise would have. He also puts himself at a risk of not having his own bid transmitted at all. Formally, his payoff from implementing this deviation strategy is,

$$\Pi^D(0; b^0) = (1 - S) P\{x < x^*\} (1 - \alpha) .$$

The dealer has no incentive to deviate if  $\Pi^D(Q; b^0) \geq \Pi^D(0; b^0)$ , or equivalently:

$$(2\alpha - 1)x^* + (1 - \alpha)\bar{x}(Q^H; I^0) \geq 0. \quad (16)$$

The existence of an  $\alpha^*$  follows from the intermediate value theorem. For  $\alpha = 0$ , expression (16) reduces to

$$-\bar{x}(S; I') + \bar{x}(Q^H; I^0) \geq 0,$$

which fails to hold under Condition 1. For  $\alpha = 1/2$ , the left-hand side of expression (16) reduces to

$$\frac{1}{2}\bar{x}(Q^H; I^0) \geq 0$$

which is true by construction.

## A.4 Proof of Proposition 1

The first part of the proof extends the derivation of  $b^0$  to the case in which  $\beta = Q^H$  and dealers bid in stage 1 with probability  $\delta$ , which may be less than unity. When dealers and fraudulent sellers bid  $Q^H$  in stage 1 with probabilities  $\delta \in [0, 1]$  and 1 respectively, then by not bidding in stage 1 and bidding only in stage 2 if  $I^0$  is reached, a collector earns:

$$\Pi^C(0; x, \delta, 1) = (1 - \alpha) P\{I^0; \delta\} [\mu\{I^0\} (1 + x) - 1] .$$

Bidding his valuation in stage 1, a collector earns:

$$\begin{aligned} \Pi^C(v(x; I^0); x, \delta, 1) &= \Pi^C(0; x, \delta, 1) / (1 - \alpha) \\ &\quad + P\{I'; \delta\} [\mu\{I'\} (1 + x) Q^H - \Delta - (1 - \Delta) S] . \end{aligned}$$

Indifference between these two actions gives rise to the derivation of  $x^*(\delta)$  in equation (7). It is straightforward to show that:

$$\frac{\partial \Pi^C(v(x; I^0); x, \delta, 1)}{\partial x} > \frac{\partial \Pi^C(0; x, \delta, 1)}{\partial x} .$$

Therefore, any collector such that  $x > x^*(\delta)$  strictly prefers to bid  $v(x; I^0)$  in stage 1 to not bidding in stage 1, then bidding only in stage 2. The rest of the proof of Lemma 2 proceeds as before.

In characterizing the best response of dealers, the proof of the result of Lemma 3 that upon bidding in stage 1, dealers bid  $Q^H$  is unaffected by allowing collectors' strategy to incorporate  $\delta < 1$ . The decision of whether to bid in stage 1 is straightforward. For any  $\delta$  that collectors input into their beliefs, which are common knowledge, a dealer



bids in stage 1 if:

$$\bar{x}(Q^H; I^0; \delta) + \alpha [x^*(\delta) - \bar{x}(Q^H; I^0; \delta)] \geq (1 - \alpha) x^*(\delta). \quad (17)$$

Since collectors' beliefs must be correct ex-post, if the inequality in (17) is strict, then  $\delta = 1$  is in equilibrium. Lastly, if (17) holds with equality at some  $\delta^*$ , then that value of  $\delta^*$  is in equilibrium as an indifferent dealer with bid in stage 1 with any probability  $[0, 1]$ . The equilibrium thus requires them to bid with probability  $\delta^*$ .

It remains to formalize the strategies of the sellers and to establish that Condition 1 is met at the uninformed seller's optimal starting price. Consider first the decision faced by the fraudulent seller. When constrained to choose actions that will not have him revealed as a fraudulent seller, he must choose a starting price of  $S$  since he is unable to mimic the starting price of an informed seller of a genuine item. When considering his stage 1 action, the choice is between a bid of  $Q^H$  and not bidding at all since any bid other than  $Q^H$  would have him revealed as a fraudulent seller. In choosing the prescribed strategy, his payoff is:

$$R^I(Q^H; b^0, \delta, 1) = (1 - x^*) + (1 - \alpha) [x^* - \bar{x}(Q^H; I^0)].$$

By not shill bidding, his payoff is:

$$R^I(0; b^0, \delta, 1) = S(1 - x^*).$$

It is sufficient that  $S \leq 1$  to establish that there is no incentive to deviate from the proposed equilibrium strategy of bidding  $Q^H$  in stage 1.

Now consider the uninformed seller's choice of starting price. When collectors bid according to  $b^0$ , dealers bid  $Q^H$  in stage 1 with probability  $\delta$  and fraudulent sellers bid  $Q^H$  in stage 1 with probability one, from the perspective of the uninformed seller,

information set  $I^0$  is reached with probability  $\gamma\mu_0$ . It is then clear that problem (8) characterizes the uninformed seller's problem. To prove the last part of the Proposition requires that the solution to problem (8) be characterized.

Let  $R^U \equiv R^U(S; \omega, b^0, \delta, 1)$  denote the uninformed seller's surplus from a starting price of  $S$ , given: valuation given by  $\omega$ , collector's bidding according to  $b^0$ , dealers bidding in stage 1 with probability  $\delta$  and fraudulent sellers bidding in stage 1 with probability one. The solution to the uninformed seller's problem depends on whether  $R^{U''} < 0$ , or equivalently, whether the following holds:

$$\gamma\mu_0 < \frac{1}{1 + \alpha}. \quad (18)$$

Since this condition is independent of  $S$ , it follows that either  $R^{U''} < 0$  or  $R^{U''} \geq 0$  in all  $S$ . The following lemma characterizes the set of possible optima, depending upon whether equation (18) holds. In doing so, it is necessary to first fix some notation. Let  $S_0$  denote the value of  $S$  such that  $c + \xi S_0 = 0$  and  $S_1$  denote the value of  $S$  such that  $c + \xi S_1 = 1$ . It follows that  $S_L = \max\{S_0, 0\}$  and  $S_U = \min\{S_1, 1\}$  represent the lower and upper bounds respectively of the range over which  $R^{U'}$  and  $R^{U''}$  are defined.

**Lemma 5** *The solution to the uninformed seller's problem,  $S^0$ , can be characterized as follows:*

1. *If  $R^{U''} < 0$ , then there exists a pair  $(\omega_L, \omega_H)$  such that:*

- *If  $\omega' \leq \omega_L$ , then  $S^0 = S_L$ ;*
- *If  $\omega' \geq \omega_H$ , then  $S^0 = 1$ ;*
- *Otherwise, there exists a unique  $S^0$  such that  $R^{U'}(S^0) = 0$ ;*

2. *If  $R^{U''} \geq 0$ , then  $S^0 = S_L$  only if  $S_L > \omega'$ ; if  $S_L \leq \omega'$   $S^0 = 1$ .*

The intuition for this result is as follows: The second-derivative of the seller's revenue function concerns the relative likelihood of information set  $I^0$  being reached. When  $I^0$  is relatively unlikely to be reached, as in when equation (18) holds, the optimal starting price focuses on the bidding of collectors, trading off, from a marginal increase in starting price, a higher sale price against a decrease in participation. Depending upon the specific parameter values, this could result in a solution in either of the corners  $\{0, 1\}$ , or an interior solution.

When  $I^0$  is sufficiently likely to be reached, the seller can guarantee herself  $Q^H$  in information set  $I^0$  by either setting the starting price to  $Q^H$  or by inducing all collectors to bid with a starting that sets  $x^*$  to zero. The difference between the two strategies is the payoff he would get in information set  $I'$ . Under the former strategy, no sale takes place in  $I'$  and he gets  $\omega'$ ; in the latter, he gets a sale at a price of  $S_L$ . Part 2 of the lemma expresses this trade-off.

The optimal starting price, when interior, satisfies the following:

$$S^0 = \frac{(1 - \gamma\mu_0)(\omega' + S_1) - \gamma\mu_0[\alpha(1 + S_0) - (1 - \alpha)\bar{x}/\xi]}{2[1 - (1 + \alpha)\gamma\mu_0]}, \quad (19)$$

where  $\bar{x} \equiv \bar{x}(Q^H; I^0; \delta)$ .

It remains to show that  $\bar{x}(Q^H; I^0) < \bar{x}(S^0; I')$ . If inequality (18) holds, then by the fact that  $S^0$  is continuous and increasing in  $\omega'$  for all  $S_L$ , the result is established using the intermediate value theorem. It is straightforward to show that  $\omega_H \leq 1$ . Therefore, if  $\omega' = 1$ , then  $S^0 = 1$ , in which case,  $\bar{x}(Q^H; I^0; \delta) < \bar{x}(Q^H; I'; \delta)$ . It follows from the intermediate value theorem that  $\omega^0 \in [0, 1]$ . If inequality (18) does not hold, then  $\omega^0 = S_L$  by Lemma 5.

## A.5 Proof of Lemma 4

Begin by considering the collector's strategy. By playing the prescribed strategy, he earns:

$$\Pi^C(b^*; x, \delta, \sigma) = \begin{cases} P\{I^0; \delta, \sigma\} [v(I^0; x; \delta, \sigma) - 1] & \text{if } x \geq \bar{x}(Q^H; \cdot) \\ + [1 - P\{I^0; \delta, \sigma\}] [v(I'; x; \delta, \sigma) - S] & \\ [1 - P\{I^0; \delta, \sigma\}] [v(I'; x; \delta, \sigma) - S] & \text{if } \bar{x}(S; \cdot) \leq x \\ & < \bar{x}(Q^H; \cdot) \\ 0 & \text{otherwise} \end{cases}$$

It should be evident that no collector type can profitably deviate to the strategy employed by any of the other collector types when the following holds:

$$\bar{x}(Q^H; I^0; 1, \sigma) \geq \bar{x}(S; I'; 1, \sigma). \quad (20)$$

Therefore, consider strategies that have the collector bid in stage 2 with positive probability. Consider then a strategy that has the collector bid in stage 1, then bid optimally in stage 2 given the information revealed in stage 1. If the collector bids in accordance with  $b^*$ , under no circumstances will he wish to bid again in stage 2. Thus, the collector would have to bid less than the amount prescribed by  $b^*$  in order for it to be profitable to bid in stage 2.

Consider first a collector for whom  $x \geq \bar{x}(Q^H; I^0; 1, \sigma)$ . By bidding less than  $v(x; I^0; 1, \sigma)$  in stage 1, then upon observing a bid in stage 1, the best he can do is to bid above  $Q^H$  in stage 2. But since his stage 2 bid will fail to transmit with positive probability, he would have been better off bidding  $v(x; I^0; 1, \sigma)$  in stage 1. Upon failing to observe a bid in stage 1, it does not matter what he had bid in stage 1 so long as it was

at least  $S$ . Thus, he is weakly better off having bid in accordance with  $b^*$ .

Now consider a collector for whom  $x \in [\bar{x}(S; I'; 1, \sigma), \bar{x}(Q^H; I^0; 1, \sigma))$ . By bidding less than  $S$ , then upon failing to observe a bid in stage 1, the best he can do is bid an amount above  $S$ . But since his bid will fail to transmit with positive probability, he would have been better off bidding some amount at least  $S$  but less than  $Q^H$ . Upon observing a bid in stage 1, it does not matter what his stage 1 bid was so long as it was less than  $Q^H$ .

## A.6 Proof of Proposition 2

The proof proceeds by taking the behavior described in the proposition as given, then showing that none of the players have an incentive to deviate from the proposed strategies.

Having established the bidding of collectors in Lemma 4, consider the dealer's strategy. By playing the prescribed strategy, he earns:

$$\Pi^D(Q^H) = (1 - S) (\bar{x}(S; I'; 1, \sigma) + \alpha [\bar{x}(Q^H; I^0; 1, \sigma) - \bar{x}(S; I'; 1, \sigma)]) .$$

By deviating to not bidding in stage 1 and bidding  $Q^H$  in stage 2, his payoffs are identical, conditional on having his bid transmitted. However, given the probability that his stage 2 bid will fail to transmit, he is strictly better off bidding  $Q^H$  in stage 1. Now suppose he deviates by bidding some amount less than  $Q^H$  in stage 1. Since any bid is indistinguishable from a bid of  $Q^H$  to a collector who bid  $S$ , this deviation will have no effect on his payoff. Such a bid may have an effect on the beliefs of a buyer who had bid in excess of  $Q^H$  as the second highest bid becomes observable to all participants. However, this bidder would have already bid more than the dealer's valuation of  $Q^H$  and it has been assumed that bids cannot be retracted. Thus, such a deviation will have no

effect on his payoff. Lastly, bidding above  $Q^H$  in either period is weakly dominated by a bid of  $Q^H$ .

Consider now the fraudulent seller's shill bidding decision. By shilling with some probability  $\sigma'$  when fraudulent sellers are thought to shill with probability  $\sigma$ , he earns:

$$R^I(\sigma'; b^*; 1, \sigma) = \sigma' [1 - \bar{x}(Q^H; I^0; 1, \sigma)] + (1 - \sigma') S [1 - \bar{x}(S; I'; 1, \sigma)].$$

Any deviation that would have the fraudulent seller bid any amount other  $Q^H$  in stage 1 would have him potentially detected as a fraudulent seller. It has been assumed that the punishment from being detected is sufficiently severe as to make such a deviation unprofitable. Thus, it is sufficient to consider deviations that have the fraudulent seller shill with some probability  $\sigma' \neq \sigma$ .

Maximizing  $R^I(\sigma'; \bar{x}(Q^H; I^0; 1, \sigma); 1, \sigma)$  with respect to  $\sigma'$ , the fraudulent seller will choose  $\sigma'$  as follows:

$$\sigma' = \begin{cases} 1 & \text{if } 1 - \bar{x}(Q^H; I^0; 1, \sigma) > S [1 - \bar{x}(S; I'; 1, \sigma)] \\ 0 & \text{if } 1 - \bar{x}(Q^H; I^0; 1, \sigma) < S [1 - \bar{x}(S; I'; 1, \sigma)] \\ \text{any } \psi \in [0, 1] & \text{if } 1 - \bar{x}(Q^H; I^0; 1, \sigma) = S [1 - \bar{x}(S; I'; 1, \sigma)] \end{cases}.$$

Thus, if  $\sigma \in (0, 1)$ , there is no incentive to deviate if

$$1 - \bar{x}(Q^H; I^0; 1, \sigma) = S [1 - \bar{x}(S; I'; 1, \sigma)].$$

If  $\sigma = 1$ , there is no incentive to deviate if

$$1 - \bar{x}(Q^H; I^0; 1, 1) \geq S [1 - \bar{x}(S; I'; 1, 1)].$$

However, if  $\sigma = 0$ , there is an incentive to deviate, except in a special case. Since

$\mu \{I^0; 1, 0\} = 1$ , it follows that  $\bar{x} (Q^H; I^0; 1, 0) = 0$ , which implies

$$1 - \bar{x} (Q^H; I^0; 1, 0) \geq S [1 - \bar{x} (S; I'; 1, 0)]$$

where the inequality is strict if  $\bar{x} (S; I'; 1, 0) > 0$ .

Given the strategies played by the aforementioned players, the uninformed seller sets the price to solve the objective function specified in the proposition. It remains to show that there exists a solution,  $S^*$ , such that equation (20) holds. In characterizing the solution to problem (13) let  $S_L = \min \{S | \bar{x} (S; I'; 1, \sigma^*) = 0\}$ . It follows that  $S_L = \mu \{I'; \sigma^*\}$ . It is evident that inequality (14) is satisfied for  $S^* = S_L$ . Thus, it remains to show that  $S_L$  is reached under certain parameterizations. There are two cases to consider, depending upon whether  $\sigma^* = 1$  or  $\sigma^* < 1$  at  $S^*$ .

First consider the case in which  $\sigma^* = 1$  at  $S^*$ . If the solution to problem (13) is interior, then  $S^*$  satisfies:

$$\gamma \mu_0 \bar{x} (Q^H; \cdot) + (1 - \gamma \mu_0) ([1 - \bar{x} (S^*; \cdot)] - (S^* - \omega') (d\bar{x} (S^*; \cdot) / dS^*)) = 0. \quad (21)$$

When  $\sigma^* = 1$  at  $S^*$ , then  $d\bar{x} (S^*; \cdot) / dS^*$  is constant in  $S^*$  and  $\sigma^*$ . It is evident from (21), that  $S^* \geq \omega'$  and that  $S^*$  is increasing in  $\omega'$ . Define  $\bar{S} = \max \{S | \bar{x} (Q^H; \cdot) = \bar{x} (S; \cdot)\}$ . It is straightforward to show that at  $\omega' = 0$ ,  $S^* < \bar{S}$ . By the intermediate value theorem, there must exist some  $\omega^*$  such that if  $\omega' = \omega^*$ , then  $S^* = \bar{S}$ .

Next, consider the case in which  $\sigma^* < 1$ . In this case, the interior optimum is as before, only now  $d\bar{x} (S^*; \cdot) / dS^* > 0$ . Therefore, it can be shown that the interior optimum under  $\sigma^* < 1$  is no greater than the interior optimum under  $\sigma^* = 1$ , all else equal. This is sufficient to prove the existence of  $\omega^*$ .

### A.7 Proof of Proposition 3

By extending the arguments from Lemma 1, it is straightforward that for a dealer to bid at all is weakly dominated by a strategy of not bidding at all. When dealers do not bid, any bid by a fraudulent seller will have him be detected as such and as such, cannot be part of an equilibrium. Consider then a collector's decision. Upon observing a starting price of at least  $Q^H$ , he correctly concludes that neither dealers nor fraudulent sellers will be in stage 1 (or any stage for that matter). Since he faces no competition from competing bidders, any strategy that has him bid in stage 2 is weakly dominated by any strategy that has him bid the stage 2 amount in stage 1 due to the probability of transmission failure. When neither dealers nor fraudulent sellers bid, the collector's beliefs conditional upon obtaining the item can be calculated from equation (11) by setting  $\delta = \sigma = 0$ , which yields beliefs of  $\mu \{I_1\}$ . Standard arguments from Lemma 1 indicate that it is weakly dominant for a collector to bid his valuation (conditional upon obtaining the item) in stage 1, which is equal to  $v(x; I_1)$ .  $v(x; I_1)$  is greater than  $S$  if and only if  $x \geq \bar{x}(S; I_1)$ .

Given a collector's best response, the uninformed seller chooses  $S'$  to solve (15). The interior optimum satisfies:

$$S' = \mu \{I_1\} + \omega(\mu_0)/2,$$

so that  $S' > 1$  if and only if

$$\mu \{I_1\} + \omega(\mu_0)/2 > 1.$$



## A.8 Proof of Proposition 4

The proof proceeds to show that there does not exist a set of beliefs upon observing a bid in stage 1 when dealers are thought to bid only in stage 2 that rationalize such an equilibrium. In such an equilibrium, fraudulent sellers must bid in stage 2 if at all. By way of contradiction, suppose that there was an equilibrium in which dealers only bid in stage 2 but fraudulent sellers bid in stage 1. Upon observing a bid in stage 1, a collector would infer that the bid was placed by a fraudulent seller and the auction would be terminated immediately.

When dealers bid in stage 2 and fraudulent sellers bid with probability  $\sigma_2 \in [0, 1]$ , collectors bid in accordance with  $b^*$ . Since both dealers and fraudulent sellers are subject to the same risk of transmission failure in stage 2, collectors' beliefs will remain those specified by (10) and (11).

Let  $\mu\{I_2\}$  denote a collector's beliefs upon observing an out-of-equilibrium bid in stage 1. If  $\mu\{I_2\} > \mu\{I_1\}$ , then collectors will increment their assessment of the item. In that case there exist collector types whose preference parameter falls within  $[\bar{x}(Q^H; I_2; \cdot), \bar{x}(Q^H; I^0; \cdot))$  that find it profitable to increment their bid of  $S$  in stage 1 to a bid of  $Q^H$ . Given a non-zero probability that such a bid will transmit, such a deviation will raise the fraudulent seller's expected revenue, while eliminating the probability that his own stage 2 bid will fail to transmit. Alternatively, if  $\mu\{I_2\} \leq \mu\{I_1\}$ , collectors will lower their assessment of the item upon observing a bid in stage 1. Unable to reduce the bids they have already placed, there will be no change in collectors' bids. Consequently, a dealer could deviate by placing a bid in stage 1 that will have no effect on the auction's outcome other than eliminate the possibility that his bid will fail to transmit. Either way, there is an incentive for either a dealer or a fraudulent seller to deviate.

## A.9 Proof of Proposition 5

The proof serves to demonstrate that the ISBE, Non-ISBE and NSBE subgames are comprehensive of all subgames following from the uninformed seller's announcement of  $S$ . Suppose  $S \leq 1$  and let  $\beta$  denote the behavioral strategy played by a dealer. Lemma 3 demonstrated that if  $\beta$  is a pure strategy, then the equilibrium has him bid  $Q^H$  in stage 1 with probability  $\delta$ . Now suppose that  $\beta$  denotes a purely mixed strategy in which  $g(\cdot)$  denotes the density of stage 1 bids over the support  $[0, 1]$ , with cdf  $G(\cdot)$ . Bids below  $S$  are not actually placed, thus  $\beta$  provides for the possibility that a dealer bids any amount and for the possibility that he bids nothing at all.

Consider the beliefs of collectors following stage 1. If no bids by opposing buyers are observed, then the collector's beliefs are given by (5) and (6) where  $\delta = 1 - G(S)$  and for some arbitrary  $\sigma$ . If a bid by an opposing buyer is observed, then the observed bid is consistent with  $\beta$  *unless* the collector is the high bidder following stage 1 *and* the standing price following stage 1 is not equal to a value in the support of  $g$ . Since any bid that a dealer would place in the support of  $g$  would lead a collector to infer that behavioral strategy  $\beta$  was being played, there is no reason for the dealer to not simply bid  $Q^H$ . This establishes the result of Lemma 3 more generally and establishes that the only degree of freedom in the dealer's strategy is over  $\delta$ .

Given the result of the previous paragraph, a fraudulent seller, upon bidding in stage 1, bids only  $Q^H$ . Thus  $\delta$  and  $\sigma$  are sufficient to characterize the strategies played by dealers and fraudulent sellers, respectively. Given  $\delta$  and  $\sigma$  and  $S$ , it is obvious that either,  $\bar{x}(Q^H; I^0; \delta, \sigma) < \bar{x}(S; I'; \delta, \sigma)$  or  $\bar{x}(Q^H; I^0; \delta, \sigma) \geq \bar{x}(S; I'; \delta, \sigma)$  is true. If the former, then the derivation of  $b^0$  has been sufficiently general that it is a best response to any  $\delta$ ,  $\sigma$  and  $S$  if  $\bar{x}(Q^H; I^0; \delta, \sigma) < \bar{x}(S; I'; \delta, \sigma)$ . However, when collectors bid according to  $b^0$ , only  $\sigma = 1$  is in equilibrium. Thus, buyers' beliefs in calculating  $\bar{x}(Q^H; I^0; \cdot)$  and  $\bar{x}(S; I'; \cdot)$  such that  $\bar{x}(Q^H; I^0; \cdot) < \bar{x}(S; I'; \cdot)$  must have  $\sigma = 1$ .

Further, for any  $\delta \in [0, 1]$ , when buyers bid according to  $b^0$ , the uninformed seller's starting price must solve problem (8). Lastly, given  $b^0$ ,  $\sigma$  and  $S$ , the derivation of  $\delta$  must be that given by Proposition 1.

If instead, collectors' beliefs are such that  $\bar{x}(Q^H; I^0; \delta, \sigma) \geq \bar{x}(S; I'; \delta, \sigma)$ . Again, the derivation of  $b^*$  has been sufficiently general so as to not depend on specific values of  $\delta$  or  $\sigma$ . As long as  $\bar{x}(Q^H; I^0; \delta, \sigma) \geq \bar{x}(S; I'; \delta, \sigma)$  is true for the  $\delta$  and  $\sigma$  assumed by collectors, no collectors will bid in stage 2 in response to the stage 1 bidding. Given that, collectors bid only in stage 1. When collectors bid only in stage 1, only  $\delta = 1$  is in equilibrium. Thus, buyers' beliefs in calculating  $\bar{x}(Q^H; I^0; \cdot)$  and  $\bar{x}(S; I'; \cdot)$  such that  $\bar{x}(Q^H; I^0; \cdot) \geq \bar{x}(S; I'; \cdot)$  must have  $\delta = 1$ . Further, for any  $\sigma \in [0, 1]$ , when buyers bid according to  $b^*$ , the uninformed seller's starting price must solve problem (13). Lastly, given  $b^*$ ,  $\delta$  and  $S$ , the derivation of  $\sigma$  must be that given by Proposition 2. This result, combined with the result of the previous paragraph, establishes that the subgame following some  $S \leq 1$  must be consistent with either the ISBE or Non-ISBE.

Consider then a starting price of  $S > 1$ . The proof of Proposition 3 is sufficient for establishing that the subgame following  $S$  must be consistent with the NSBE.

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